MATH 161 SOLUTIONS: QUIZ III 20 SEPTEMBER 2019

1. Marcel, a Math 161 student, realizes that the more caffeine he consumes, the faster he completes his WebAssign homework. Before starting tonight's assignment, he buys a cup of coffee containing a total of 100 milligrams of caffeine. Let $T(c)$ be the number of minutes it will take Marcel to complete tonight's assignment if he consumes c milligrams of caffeine. Suppose that T is continuous and differentiable.

a) *[1 pt]* What are the units of T' ?

Answer: minutes / milligram

b) *[1 pt]* Circle the one sentence below that is best supported by the statement "the more caffeine Marcel consumes, the faster he completes his online homework assignments."

i. $T'(c) \geq 0$ for every value c in the domain of T.

ii. $T'(c) \leq 0$ for every value c in the domain of T.

iii. $T'(c) = 0$ for every value c in the domain of T.

Solution: *As the amount of caffeine consumed, c, by Marcel increases, the time needed for him to complete tonight's assignment decreases!*

c) *[1 pt]* Interpret the equation $T^{-1}(100) = 45$ in the context of this problem. Use a complete sentence and include units.

Solution: In order for Marcel to complete his homework assignment in 100 minutes, he must consume 45 milligrams of caffeine.

2. *[4 pts]* Let $f(x) = \frac{x}{x^2+1}$ at x = 2. Your trustworthy friend, Albertine, tells you that $f'(x) = \frac{1-x^2}{(1+x^2)^2}$.
Write an equation of the tangent line to y = f(x) at x = 2.

Solution: The point of tangency is P = (2, f(2)) = (2, 2/5).

The slope of the tangent line at x = 2 is $f'(2) = \frac{1-2^2}{(1+2^2)^2} = -\frac{3}{25}$.

Thus an equation of the tangent line is:

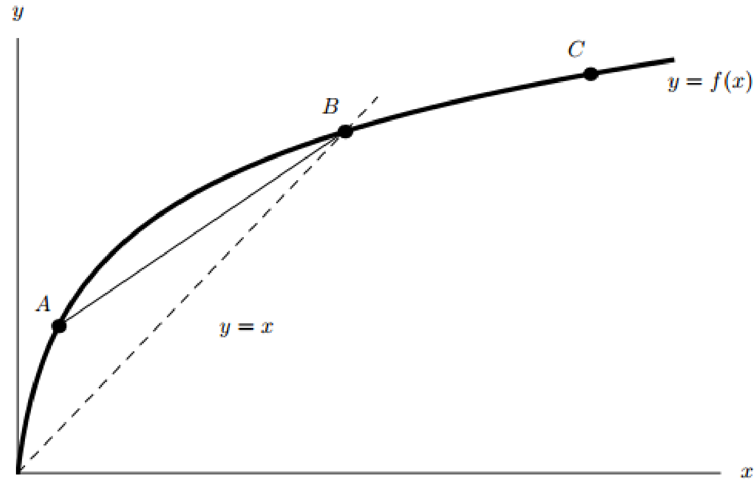
$$y - \frac{2}{5} = -\frac{3}{25}(x-2)$$

3. *[5 pts]* For the graph of y = f (x) in the figure below, arrange the following numbers from ***smallest to largest:***

A The slope of the curve at A.
B The slope of the curve at B.
C The slope of the curve at C.
AB The slope of the line *AB*.

0 The number 0.
1 The number 1.

Explain the positions of the number 0 and the number 1 in your ordering. Any unclear answers will not receive credit.



0 < **C** < **B** < **AB** < **1** < **A**

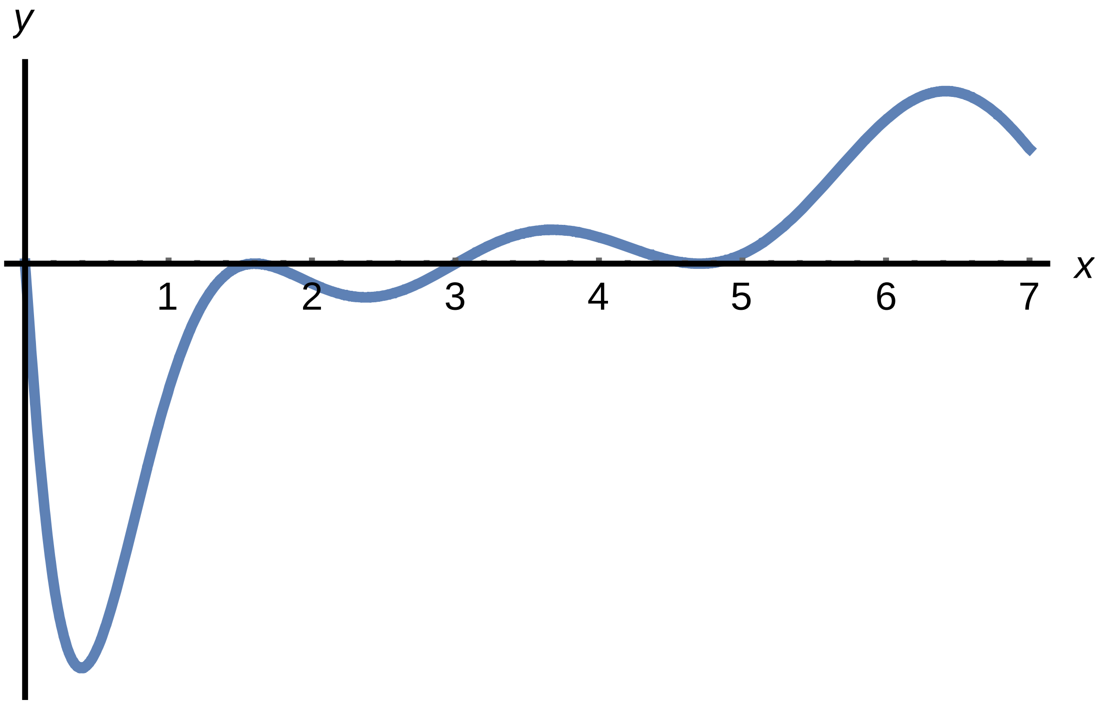
Solution: The number one and all other slopes are positive, so 0 must be the smallest number. The line $y = x$ has a slope of 1. The slope at C, the slope at B, and the slope of the line AB are each smaller than the slope of the line $y = x$ by looking at the picture. The slope at A is larger than the slope of $y = x$ also by the picture. Thus 1 is the second to largest number in the ordering.

4. *[7 pts]* Using the *limit definition* of derivative find the (numerical value of the) slope of the curve $g(x) = 3x^2 - 5x + 7$ *at the point* $x = -1$. Show all steps!

Solution: $g'(-1) = \lim_{h\to 0} \frac{g(-1+h)-g(-1)}{h} = \lim_{h\to 0} \frac{(3(-1+h)^2-5(-1+h)+7)-(3(-1)^2-5(-1)+7)}{h} =$
$\lim_{h\to 0} \frac{3h^2-11h}{h} = \lim_{h\to 0} \frac{h(3h-11)}{h} = \lim_{h\to 0} (3h-11) = -11$

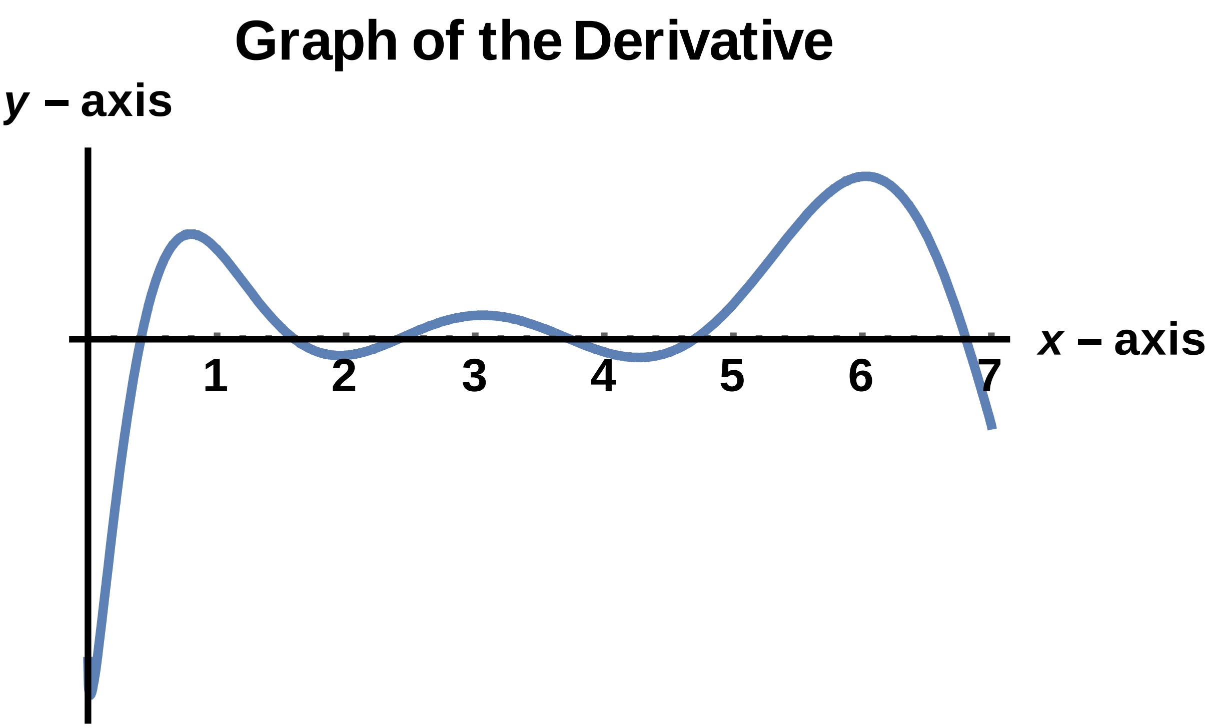
5. *[6 pts]* Using the process of "geometric differentiation," sketch the graph of the derivative of the function y = G(x) whose graph is given below.

FYI: This is the graph of $y = x(x-1.6)^2(x-3)(x-4.7)^2(x-8)^2(1+\sqrt{x})e^{-x/8}$



c

Solution: Begin by finding the zeroes of dy/dx by looking for critical points (horizontal tangent lines) in the graph of y = f(x). Then perform a sign analysis on dy/dx by looking for regions of increase and decrease in y = f(x).



6. ***Extra credit*** *[5 pts]* Suppose that W(h) is an increasing function which tells us how many gallons of water an oak tree of height *h* feet uses on a hot summer day.

(a) Give *practical interpretations* for each of the following quantities or statements. *Use a complete sentence for each with no technical jargon.*

- $W(50)$

Solution: The expression $W(50)$ represents how many gallons of water a 50 foot tall oak tree uses on a hot summer day.

- $W^{-1}(40)$

Solution: The expression $W^{-1}(40)$ represents the height of an oak tree (in feet) which uses 40 gallons of water on a hot summer day.

- $W'(5) = 3$

Solution: An oak tree which is 6 feet tall uses approximately 3 more gallons of water per hot summer day than a 5 foot tall oak tree does.
OR
If a 5 foot tall oak tree grew an extra foot, it would use approximately 3 more gallons of water per hot summer day.

(b) Suppose that an *average oak tree is A feet tall and used G gallons* of water on a hot summer day. Answer each of the questions below in terms of the function *W*. You may also use the constants *A* and/or *G* in your answers.

- A farmer has a grove with 25 oak trees, and each one is 10 feet taller than an average oak tree. How much water will be used by her trees during a hot summer day?

Solution: $25W(A + 10)$ gallons

- The farmer also has some oak trees that use 5 fewer gallons of water on a hot summer day than an average oak tree does. How tall is one of these trees?

Solution: $W^{-1}(G - 5)$ feet

The only time my education was interrupted was when I was in school.

- George Bernard Shaw