MATH 161 SOLUTIONS: QUIZ IV (EXTRA CREDIT)

11 OCTOBER 2019

1. *[4 pts each]* Find an *anti-derivative* of each of the following functions:

(a) $\frac{2+x^7}{x^9}$

Solution: $\frac{2+x^7}{x^9} = 2x^{-9} + x^{-2}$

An excellent first guess might be $x^{-8} + x^{-1}$

Correcting for constants the answer is $\boldsymbol{-\frac{1}{4}\, x^{-8} - x^{-1}}$

(b) $3e^x + 4\sec^2 x + \pi$

Solution: A good first guess might be $e^x + \tan x + \pi x$.

Correcting for multiplicative constants, the answer is $\boldsymbol{3e^x + 4\ \tan x + \pi x}$

(c) $(\cos x)\ (\tan x)$ *Hint: try to rewrite this function before seeking an anti-derivative*

Solution: First note that $\cos x \tan x = \cos x\, \frac{\sin x}{\cos x} = \sin x$

Thus an antiderivative that we seek is $\boldsymbol{-\cos x}$

2. *[4 pts each]* Use the chain rule compute the derivative of each of the following functions.

You *need not* simplify. (*Remember:* Parentheses are important if not essential!)

(a) e^{1+x^3}

Solution: Using the chain rule (shortcut):

$$\frac{d}{dx} e^{1+x^3} = e^{1+x^3} \frac{d}{dx}(1 + x^3) = \boldsymbol{3x^2 e^{1+x^3}}$$

(b) $\tan(\sin x)$

Solution:

Using the chain rule (shortcut):

$$\frac{d}{dx}\tan(\sin x) = sec^2(\sin x)\ \frac{d}{dx}(\sin x) = \boldsymbol{(\cos x)\ sec^2(\sin x)}$$

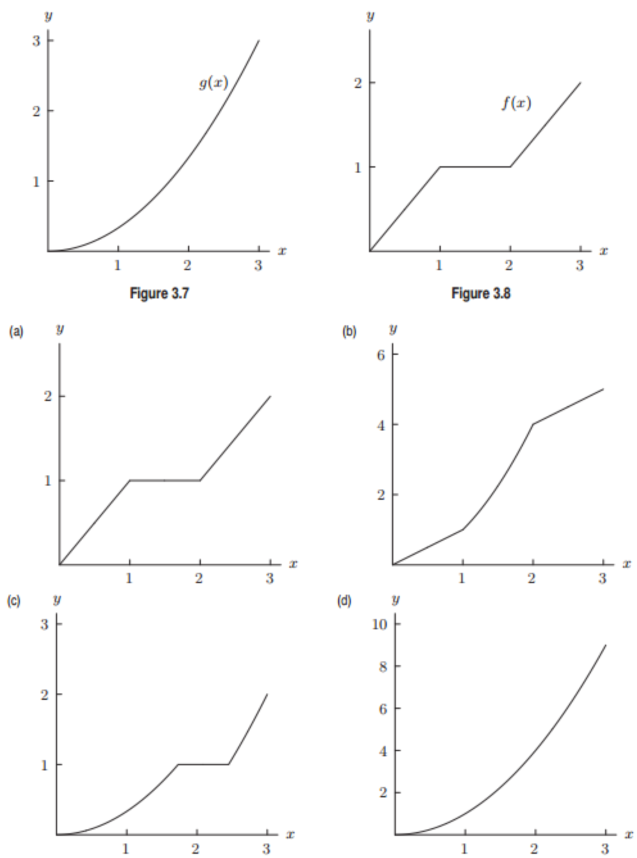
(c) $(1 + x + x^3)^{2019}$

Solution: Using the general power rule,

$$\frac{d}{dx}(1 + x + x^3)^{2019} = 2019(1 + x + x^3)^{2018}\frac{d}{dx}(1 + x + x^3) =$$

$$2019(1 + x + x^3)^{2018}(1 + 3x^2) = \mathbf{2019(1 + 3x^2)(1 + x + x^3)^{2018}}$$

3. *[1 pt]* Given the graphs of the functions f(x) and g(x) in figures 3.7 and 3.8, which of the following (a) – (d) is a graph of $f \circ g(x)$? (You need not explain how you chose your answer.)



Solution: ***C*** **is the correct choice**

Here is one of many reasons:

Using the chain rule, (f(g(x)))′ = f ′(g(x))g′(x), we see f(g(x)) has a horizontal tangent whenever g′(x) = 0 or f ′(g(x)) = 0. Now, f ′(g(x)) = 0 for 1 < g(x) < 2 and this approximately corresponds to 1.7 < x <2.5.

4. *[8 pts]* Determine *concavity and inflection points (if any)* of the function

$$f(x) = x^4 - 4x^3 + 2019$$

Express your answers in interval form. *You need not graph the curve!*

Solution: The solution requires that we find the second derivative of f and then perform a sign analysis on it. Towards this end:

$$f'(x) = 4x^3 - 12x^2$$

$$f''(x) = 12x^2 - 24x \;=\; 12x(x-2)$$

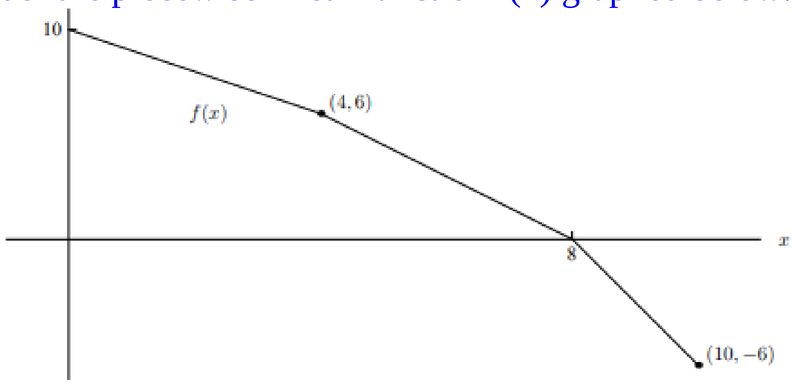
In performing a sign-analysis of f'' we note that the transition points are x = 0 and x = 2.

Moreover, $f'' > 0$ *on* $(-\infty, 0)$ *and on* $(2, \infty)$. Also, $f'' < 0$ *on* $(0, 2)$.

Thus f *is concave up on* $(-\infty, 0)$ *and on* $(2, \infty)$; f *is concave down on* $(0, 2)$.

So there are 2 **points of inflection: at x = 0 and at x = 2.**

5. *[4 pts each]* Consider the piecewise linear function f(x) graphed below:



For each function g (x), find the value of $g'(3)$.

(a) $g(x) = \sin\left(\left(f(x)^3\right)\right)$ (Here you need to use the chain rule twice.)

Solution: Using the chain rule:

$$g(x) = \frac{d}{dx}\sin\left((f(x)^3)\right) = \cos\left((f(x)^3)\right)\frac{d}{dx}(f(x)^3) =$$

$$\cos\left((f(x)^3)\right)\frac{d}{dx}(f(x)^3) = \cos\left((f(x)^3)\right)3(\mathrm{f(x)})^2 f'(x)$$

Now: Since the slope of the curve at x=3 is $\frac{6-0}{4-10} = -1$, the equation of the curve in the vicinity of x = 3 is

f(x) = -1(x) + 10.

So f(3) = 7 *and* $f'(3) = -1$.

Hence $g'(3) = \cos\left((f(3)^3)\right)3(\mathrm{f}(3))^2 f'(3) =$

$$\cos(7^3)3(7)^2(-1) = -147\cos(7^3) \cong \mathbf{124.04}$$

(b) $g(x) = \frac{f(x^2)}{x}$ (Here you must use the quotient rule followed by the chain rule.)

Solution: Using the quotient rule,

$$g'(x) = \frac{x\frac{d}{dx}f(x^2) - f(x^2)\frac{d}{dx}(x)}{x^2}$$

Invoking the chain rule to compute $\frac{d}{dx}f(x^2)$, *we find that*

$$g'(x) = \frac{x\left(f'(x^2)\right)2x - f(x^2)}{x^2}$$

And so

$$g'(3) = \frac{3(f'(9))2(3) - f(9)}{9}$$

Since f(9) = -3 and $f'(9) = -3$, *we have*:

$$g'(3) = \frac{3(-3)2(3) - (-3)}{9} = -\frac{17}{3} \cong \mathbf{-5.67}$$

DERIVATIVE RULES

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}\left(a^x\right) = \ln a \cdot a^x \qquad \frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\cdot g'(x) + g(x)\cdot f'(x) \qquad \frac{d}{dx}(\sec x) = \sec x\tan x \qquad \frac{d}{dx}(\csc x) = -\csc x\cot x$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2} \qquad \frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(f(g(x))) = f'(g(x))\cdot g'(x) \qquad \frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

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O dear Ophelia!
I am ill at these numbers:
I have not art to reckon my groans.

- HAMLET (Act II, Sc. 2)