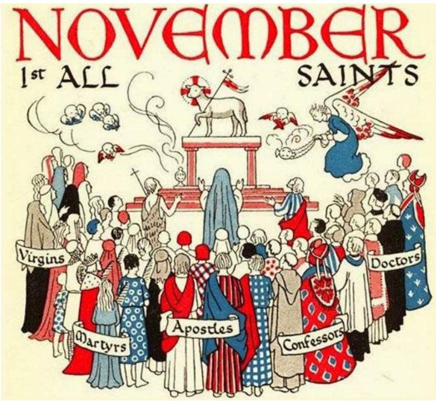
MATH 161 SOLUTIONS: QUIZ VI 1 NOVEMBER 2019



Instructions: Answer *any 3 of the following 4 problems*. You may answer all 4 to earn extra credit.

1. Suppose that the side of a huge cube of ice is *decreasing* at a rate of 0.5 cm/min when the side has length 13 cm. At what rate is the *surface area* of the cube changing at that moment?

- *Remember to define your variables, include units, draw a diagram, state what is given, and what your goal is.*

Solution:

Let *x* = side of cube (in cm). Let S = surface area of the cube (in cm^2).

Given: *dx/dt = -0.5 when x = 13.*

Find: *dS/dt when r = 8*

Since S = f(x) and x = g(t), we can invoke the Chain Rule:

$$\frac{dS}{dt} = \frac{dS}{dx}\frac{dx}{dt}$$

Since a cube has 6 faces, S = $6x^2$. Then dS/dx = 12x.

Substituting in the Chain Rule above:

$$\frac{dS}{dt} = (12x)\,(-0.5)$$

When x = 13,

$$\frac{dS}{dt} = 12(13)(-0.5)\ = \mathbf{-78\ cm^2/min}$$

2. Harry, the potter, has a *fixed* volume of clay (28.3 cm^3) in the form of a cylinder. As he rolls the clay, the length of the cylinder, *L*, increases, while the radius, *r*, decreases. If the length of the cylinder is increasing at a constant rate of 0.2 cm per second, find the rate at which the radius is changing when the radius is 1.5 cm, and the length is 4 cm. *(You need not simplify your numerical answer.)*

- *Remember to define your variables, include units, draw a diagram, state what is given, and what your goal is.*

Solution:

We are given that dV/dt = 0 (since the potter has a fixed volume of clay).

Also, we are given that dh/dt = 0.2.

We know that $V = \pi r^2 h$. Since each of r and h is a function of time, we may differentiate implicitly to obtain:

$$0 = dV/dt = \pi\{ r^2 (dh/dt) + 2rh\,(dr/dt)\}.$$

Thus $0 = 0.2\, r^2 + 2rh\,(dr/dt)$.

When r = 1.5 and h = 4, we find that:

$$0 = 0.2(1.5)^2 + 2(1.5)(4)\,(dr/dt)$$

Hence dr/dt = -1.5/8 = ***-0.0375 cm/sec***

3. Albertine and Jean-Luc were friends in high school but then went to college in different parts of the country. They expected to meet in Detroit during the December break, but their schedules didn't match up. It turns out that Albertine is leaving on the same day that Jean-Luc is arriving.
Shortly before Jean-Luc's train arrives in Detroit, he sends a text to Albertine to see where she is, and Albertine sends a text response to say that, sadly, her train has already left. At the moment Albertine sends her text, she is 20 miles due east of the center of the train station and moving east at 30 mph while Jean-Luc is 10 miles due south of the train station and moving north at 50 mph.

(a) What is the distance between Jean-Luc and Albertine at the time Albertine sends her text?

- *Remember to define your variables, include units, draw a diagram, state what is given, and what your goal is.*

Solution:

Let x, y, z be the distances (in miles) between Albertine and the station, Swann and the station, and Albertine and Swann, respectively.

Then, since the triangle is a right triangle, $x^2 + y^2 = z^2$.

When Albertine sends her text, x = 20 and y = 10. So Albertine and Swann are

$$\sqrt{20^2 + 10^2} = 10\sqrt{5} \approx \mathbf{22.4\ miles\ apart}.$$

(b) When Albertine sends her text, are she and Jean-Luc moving closer together or farther apart? How quickly? *(You need not simplify your numerical answer.) You must show your work clearly to earn any credit. Remember to include units.*

Solution:

Differentiating $x^2 + y^2 = z^2$ implicitly (with respect to t) yields:

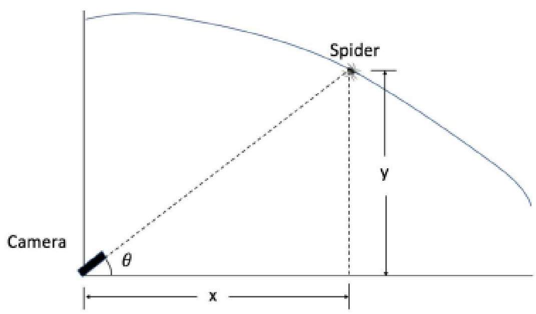
2x (dx/dt) + 2y (dy/dt) = 2z(dz/dt)

When Swann sends his text, we know that dx/dt = 30 and dy/dt = -50. Hence

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z} = \frac{(20)(30)+(10)(-50)}{10\sqrt{5}} = 2\sqrt{5} \approx 4.47$$

Since, at this time, the sign of dz/dt is positive, Albertine and Swann are moving ***farther apart*** *at a rate of approximately* ***4.47 miles/hr.***

4. *(University of Michigan)* Albertine is making a documentary video about the wildlife that lives in a local cave. She found a spider of a new species climbing down along the ceiling of the cave (as shown in the diagram below).



Here

- x is the spider's distance to the right, in ft, of the camera
- y is the height, in ft, of the spider from the ground
- θ is the angle, in radians, made by the ground and the line joining Albertine's camera and the spider.

The camera is following the spider as it walks along the ceiling of the cave. Find the rate at which the angle θ is changing when the following conditions hold:

- The spider is 10 ft above the ground.
- The spider's distance to the right of the camera is increasing at 0.4 feet per second.
- The spider's height is decreasing at a rate of 0.2 feet per second.
- The angle $\theta = \frac{\pi}{6}$ $radians$.

Hint: Use the equation $\tan\theta = \frac{y}{x}$ satisfied by the variables x, y and θ to find your answer. Include units. Show all your work.

Solution: Taking derivatives of $\tan\theta = \frac{y}{x}$ with respect to time we obtain

$$(\sec^2\theta)\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}\ .$$

Solving for $\frac{d\theta}{dt}$ we have

$$\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}(\cos^2\theta)\ . \quad \text{(Eqn *)}$$

We are given $y = 10,\ \frac{dx}{dt} = 0.4,\ \frac{dy}{dt} = -0.2\ and\ \theta = \frac{\pi}{6}$.

Then $\tan\theta = \frac{y}{x}\ yields\ \frac{10}{x} = \frac{1}{\sqrt{3}}\ which\ implies\ that\ x = 10\sqrt{3}$.

Substituting these values into equation (*) yields

$$\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}(\cos^2\theta) = \frac{10\sqrt{3}(-0.2) - 10\ (0.4)}{\left(10\sqrt{3}\right)^2}\left(\cos^2\frac{\pi}{6}\right) = \frac{-2\sqrt{3}-4}{300}\left(\frac{3}{4}\right) = \frac{-\sqrt{3}-2}{200}\ \boldsymbol{radians\ /second}.$$

The only way to learn mathematics is to do mathematics. That tenet is the foundation of the do-it-yourself, Socratic, or Texas method.

- Paul Halmos