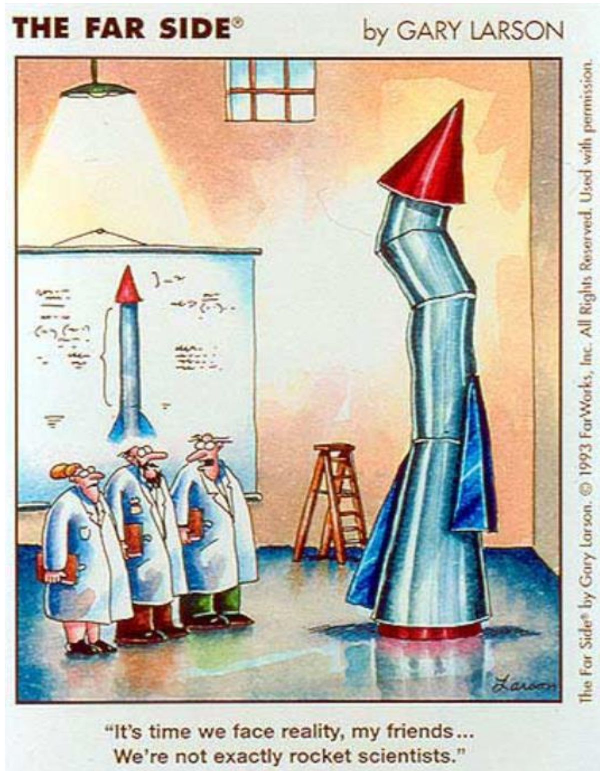
MATH 161 PARTIAL SOLUTIONS: TEST II 18 OCTOBER 2019

Instructions: Answer any ***9*** *of the following* ***11*** *questions. You may answer more than 9 to earn extra credit. Each problem is worth 12 pts.*



"It's time we face reality, my friends... We're not exactly rocket scientists."

1. Differentiate each of the following functions. You *need not* simplify. Show your work.

(a) $y = x^3\cos(4x)$

Solution: Using the product rule first,

$$\frac{dy}{dx} = x^3\frac{d}{dx}\cos(4x) + \cos(4x)\,\frac{d}{dx}\,x^3 = x^3\frac{d}{dx}\cos(4x) + 3x^2$$

Next, using the chain rule,

$$x^3(-\sin(4x))\frac{d}{dx}(4x) + 3x^2 = \mathbf{-4x^3\sin(4x) + 3x^2}$$

(b) $y = \mathrm{x}\,e^{4x} - 5x + \sqrt[3]{2019}$

Solution: Using the linearity of the derivative,

$$\frac{dy}{dx} = \frac{d}{dx}(\mathrm{x}\,e^{4x}) - \frac{d}{dx}5x + \frac{d}{dx}\sqrt[3]{2019}$$

Next, using the product rule and recalling that the derivative of a constant is 0, we obtain,

$$\frac{d}{dx}(\mathrm{x}\, e^{4x}) - \frac{d}{dx}5x + \frac{d}{dx}\sqrt[3]{2019} =$$

$$e^{4x}\frac{d}{dx}(\mathrm{x}\,) + x\frac{d}{dx}(e^{4x}) - 5 + 0 =$$

$$e^{4x} + x(4e^{4x}) - 5 =$$

$$\boldsymbol{e^{4x} + 4xe^{4x} - 5}$$

(c) $\sin(\tan(x^4 + 1))$

Solution: Using the chain rule twice,

$$\frac{d}{dx}\sin(\tan(x^4 + 1)) = \cos(\tan(x^4 + 1))\frac{d}{dx}\tan(x^4 + 1) =$$

$$\cos(\tan(x^4 + 1))\frac{d}{dx}\tan(x^4 + 1) =$$

$$\cos(\tan(x^4 + 1))\sec^2(\tan(x^4 + 1))\frac{d}{dx}(x^4 + 1) =$$

$$\cos(\tan(x^4 + 1))\sec^2(\tan(x^4 + 1))4x =$$

$$\mathbf{4x\cos(\tan(x^4 + 1))\sec^2(\tan(x^4 + 1))}$$

2. Consider the curve $\mathrm{f(x) = (2x - 1)^{75}(x + 8)^{74}}$.

(a) Find *all* critical points of f(x).

Solution:

$$F'(x) = (2x - 1)^{75}(x + 8)^{74} =$$

$$(2x - 1)^{75}\frac{d}{dx}(x + 8)^{74} + (x + 8)^{74}\frac{d}{dx}(2x - 1)^{75} =$$

$$(2x - 1)^{75}\, 74(x + 8)^{73} + (x + 8)^{74}\,(75(2x - 1)^{74})2 =$$

$$((2x - 1)^{74}\,(x + 8)^{73})\{74(2x - 1) + 150(x + 8)\} =$$

$$2(2x - 1)^{74}\,(x + 8)^{73}(149x + 563)$$

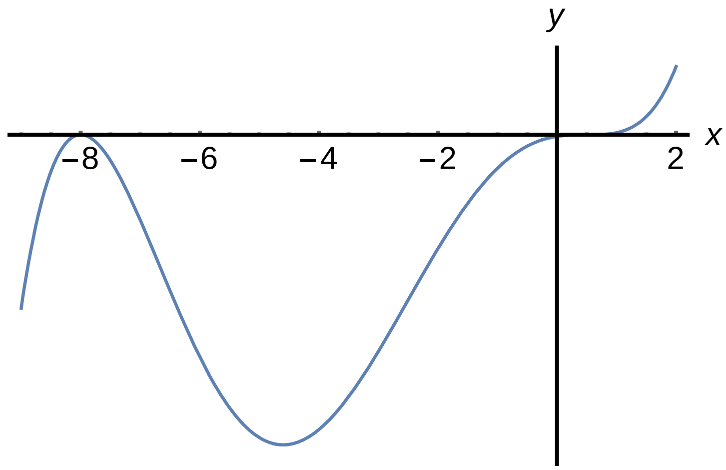
Hence the critical points are **x = ½, x = -8, and x = -563/149**

(b) Determine the intervals upon which the graph of y = f(x) is rising and those where it is falling. Now classify each critical point.

Solution:

Rising on (-∞ . -8) and on (-563/149, ∞)

Falling on (-8, -563/149)



3. Albertine is lost in Death Valley, and she needs to build a cube out of cactus skins to hold her precious supplies. She wants her cube to have a volume of 8.1 cubic feet, but she needs to figure out the side length to cut the cactus skins the right size. She has forgotten her beloved calculator, so she decides to figure out the side length of her cube using calculus.

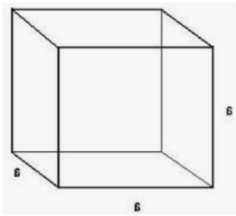
(a) Find the *local linearization* of the function $f(x) = \sqrt[3]{x}$ at x = 8.

Solution:

(b) Use your linearization to approximate $\sqrt[3]{8.1}$

Solution:

(c) Is your answer an over- or under-estimate? Explain. (Use of calculator here is forbidden.)



Solution:

4. Using the method of *judicious guessing*, find an *antiderivative* for each of the following functions. *Be certain to show your reasoning! Circle your final answer!*

(a) $x + x^{\frac{1}{2}} + x^{-3}$

Solution: $\frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}\, x^{-2}$

(b) $\frac{3\cos x + 5\, cos^2 x + 7\sec x}{\cos x}$

Solution:

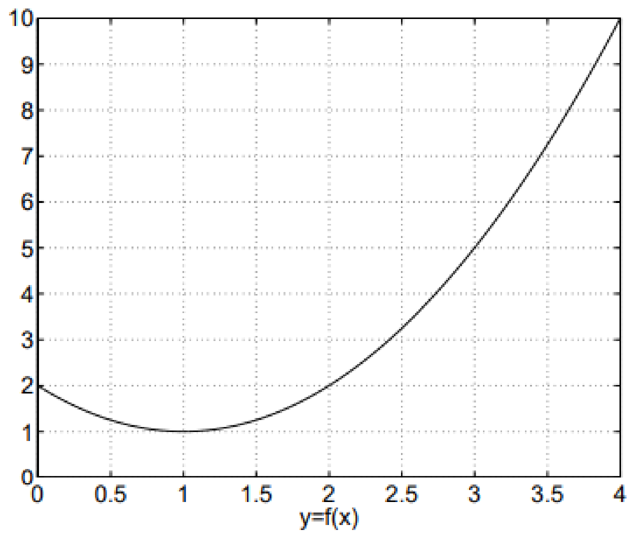
(c) $e^{3x} + \cos 5x$

Solution:

(d) $x\, sec(x^2)(tan(x^2))$

Solution:

5. Let *f* and *g* be functions with the following graphs:



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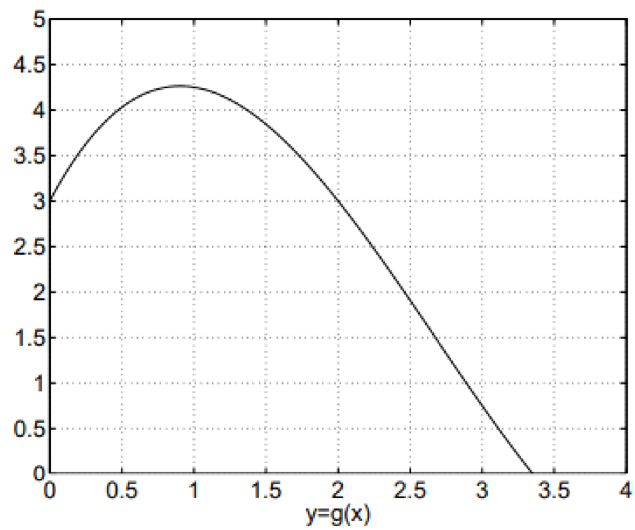


Use the above graphs to *estimate* each of the following derivatives. *Show your work* and circle your answers.

(a) $h'(2)\ if\ \ h(x) = f(x)g(x)$

Solution:

(b) $h'(2)\ if\ \ h(x) = \frac{f(x)}{g(x)}$

Solution:

(c) $h'(2)\ if\ \ h(x) = f(g(x))$

Solution:

6. Let $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - 3x^2 + 1$

(a) Find and classify all the critical points of *f*. *Justify your answers.*

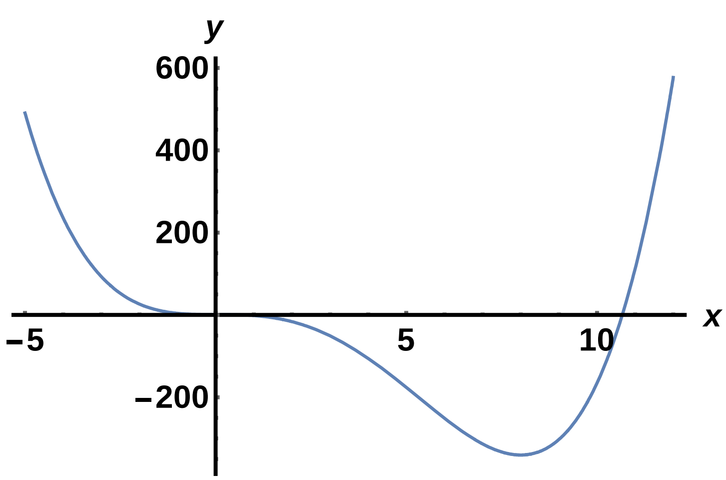
Solution:

(b) Find all points of inflection of *f*. *Justify your answer*.

Solution:

(c) Based upon the information obtained in (a) and (b), sketch the graph of y = f(x). Employ all three stages of curve sketching. *Note that since f(x) is not factorable, we must dismiss most of State I (limiting behavior remains.)*

Solution:



7. Let $F(x) = \frac{x^3 - 1}{x^2}$

(a) Sketch the curve using only Precalculus (that is, Stage 1).

Solution:

(b) Find and classify all critical points using the first derivative test.

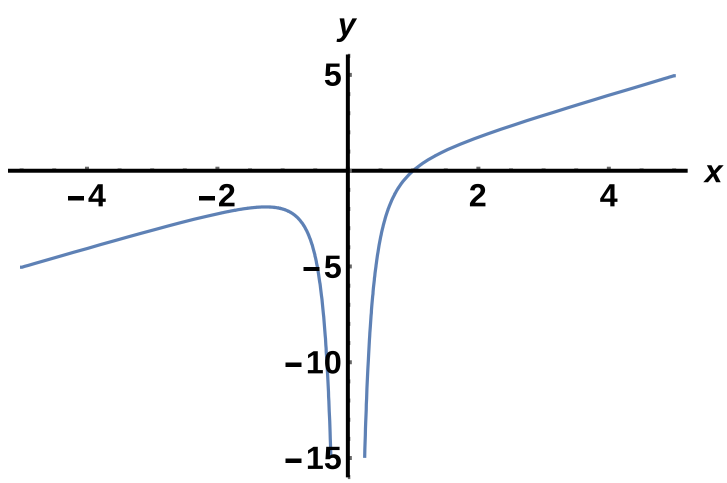
Solution:

(c) Find the points of inflection.

Solution:

(d) Now sketch an improved version of y = F(x), using Stages II and III.

Solution:



8. Let $G(x) = x^3 e^{2x}$

(a) Sketch the curve using only Precalculus techniques.

Solution:

(b) Find and classify all critical points using the first derivative test.

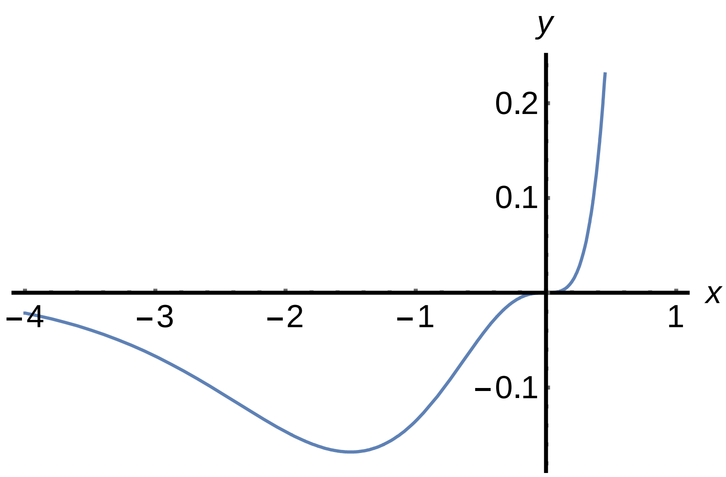
Solution:

(c) Find the points of inflection.

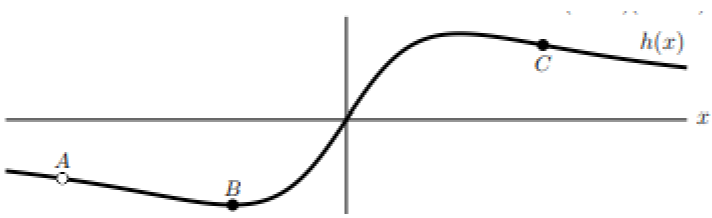
Solution:

(d) Now sketch an improved version of y = F(x). Label any and all local extrema and inflection points.

Solution:



9. Below is a graph of the function h(x) defined by $h(x) = \frac{2x^2+10x}{(x+5)(x^2+4)}$



(a) The point **A** is a hole in the graph of *h*. Find the x- and y-coordinates of **A**.

Solution: Simplifying h(x), we obtain $h(x) = \frac{2x(x+5)}{(x+5)(x^2+5)}$. Since the factor (x + 5) cancels, the hole at x = -5 is actually a removable discontinuity).. We compute4 the limit of h(x) as $x \to -5$ to find the y-coordinate, viz.

$$\lim_{x\to -5} h(x) = \lim_{x\to -5} \frac{2x}{x^2+4} = -\frac{10}{29}$$

$$Thus\ \boldsymbol{A} = \left(-\mathbf{5}, -\frac{\mathbf{10}}{\mathbf{29}}\right)$$

(b) The point **B** is a local minimum of *h*. Find the x- and y-coordinates of **B**.

Solution:
Using the quotient rule on the simplified form of h, we have

$$h'(x) = \frac{d}{dx}\left(\frac{2x}{x^2+4}\right) = \frac{(x^2+4)2 - 2x(2x)}{(x^2+4)^2} = 2\frac{4-x^2}{(x^2+4)^2}$$

This is never undefined, and it is equal to zero when $4 - x^2 = 0$.

Thus $x = \pm 2$ are critical points. Now B has a negative x-coordinate, thus x = -2.

The y-coordinate is $y = \frac{-4}{8} = -\frac{1}{2}$. $Thus\ B = \left(-2, -\frac{1}{2}\right)$

(c) The point **C** is an inflection point of *h*. Find the x- and y-coordinates of **C**.

Solution:

We use the quotient rule again to find

$$h''(x) = \frac{2x^3 - 24x}{(x^2+4)^3} = \frac{2x(x^2-12)}{(x^2+4)^3}$$

This is never undefined, and it is zero when $2x(x^2 - 12)$, i.e. when $x = 0, \pm 2\sqrt{3}$

From the graph, we see that our x-coordinate of C must be $2\sqrt{3}$, and then y $= \frac{4\sqrt{3}}{16} = \frac{\sqrt{3}}{4}$.

Hence $C = \left(2\sqrt{3}, \frac{\sqrt{3}}{4}\right)$.

10. Let $f(x) = x^4 - ax^2$.

(a) Find all possible critical points of *f* in terms of *a*.

Solution:

(b) If $a < 0$, how many critical points does *f* have?

Solution:

(c) If $a > 0$, find the *x* and *y* coordinates of all critical points of *f*.

Solution

(d) Find a value of *a* such that the two local minima of *f* occur at $x = \pm 2$.

Solution

11. Find the values of constants *a*, *b*, and *c* so that the graph of

$y = \frac{x^2+a}{bx+c}$ has a local *minimum* at x = 3 and a local *maximum* at (-1, -2).

Solution