MATH 201: CLASS DISCUSSION

29 AUG 2019

INTRODUCTION TO SETS

STUDY CAREFULLY SECTIONS 1.1 – 1.4 OF HAMMACK

The following notation is commonly used: **Z** for the set of integers; **N** for the set of positive integers (the "natural numbers"), **Q** for the set of all rational numbers; **R** for the set of all real numbers, and ∅ the empty set.

A. List the elements of each of the following sets: (a) $\{x \in R : x^4 - 1 = 0\}$ (b) $\{x \in Z : -1/3 < x < 5.99\}$

(c) $\{x \in N \mid x \leq 4\}$ (d) {unicorns| unicorn lives in Illinois} (e) {} (f) {φ}

(g) {1, {2}}

B. Write in set notation: (a) {4, 9, 16, 25, …} (b) {1/1, 1/3, 1/5, 1/7, …} (c) {…l/8, ¼, ½, 1, 2, 4, 8, …}

C. Determine the cardinality of each set in (A).

D. Find cardinality of each of the following sets:

$\{Kansas\ City, Phoenix, Sacramento, Denver\}$

{{Friday}, {blue moon}, {dragonfly}}

{{1, 2}, {{7, 0, 3}}}

$\{p \in N \mid p\ is\ prime\ and\ p \leq 25\}$

{{1}, {2, {3, 4}}, ∅}

{{1, 4}, a, b, { {3, 4}}, {∅}}

$\{x \in Z \mid |x| < 10\}$

E. Sketch the following sets of points in the xy-plane.

$\{(x,y) : x, y \in \mathbb{R}, x^2 + y^2 \leq 1\}$

$\{(x,y) : x, y \in \mathbb{R}, y \geq x^2 - 1\}$

$\{(x,y) : x, y \in \mathbb{R}, x > 1\}$

$\{(x, x+y) : x \in \mathbb{R}, y \in \mathbb{Z}\}$

$\{(x, \frac{x^2}{y}) : x \in \mathbb{R}, y \in \mathbb{N}\}$

$\{(x,y) \in \mathbb{R}^2 : (y-x)(y+x) = 0\}$

$\{(x,y) \in \mathbb{R}^2 : (y-x^2)(y+x^2) = 0\}$

F.

1. Suppose $A = \{1,2,3,4\}$ and $B = \{a,c\}$.

(a) $A \times B$ (c) $A \times A$ (e) $\emptyset \times B$ (g) $A \times (B \times B)$
(b) $B \times A$ (d) $B \times B$ (f) $(A \times B) \times B$ (h) B^3

2. Suppose $A = \{\pi,e,0\}$ and $B = \{0,1\}$.

(a) $A \times B$ (c) $A \times A$ (e) $A \times \emptyset$ (g) $A \times (B \times B)$
(b) $B \times A$ (d) $B \times B$ (f) $(A \times B) \times B$ (h) $A \times B \times B$

3. $\{x \in \mathbb{R} : x^2 = 2\} \times \{a,c,e\}$
4. $\{n \in \mathbb{Z} : 2 < n < 5\} \times \{n \in \mathbb{Z} : |n| = 5\}$
5. $\{x \in \mathbb{R} : x^2 = 2\} \times \{x \in \mathbb{R} : |x| = 2\}$
6. $\{x \in \mathbb{R} : x^2 = x\} \times \{x \in \mathbb{N} : x^2 = x\}$
7. $\{\emptyset\} \times \{0,\emptyset\} \times \{0,1\}$
8. $\{0,1\}^4$

Sketch these Cartesian products on the x-y plane $\mathbb{R}^2$ (or $\mathbb{R}^3$ for the last two).

9. $\{1,2,3\} \times \{-1,0,1\}$
10. $\{-1,0,1\} \times \{1,2,3\}$
11. $[0,1] \times [0,1]$
12. $[-1,1] \times [1,2]$
13. $\{1,1.5,2\} \times [1,2]$
14. $[1,2] \times \{1,1.5,2\}$
15. $\{1\} \times [0,1]$
16. $[0,1] \times \{1\}$
17. $\mathbb{N} \times \mathbb{Z}$
18. $\mathbb{Z} \times \mathbb{Z}$
19. $[0,1] \times [0,1] \times [0,1]$
20. $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 1\} \times [0,1]$

G. Find the power set of each of the following sets:

1. $\{1,2,3,4\}$
2. $\{1,2,\emptyset\}$
3. $\{\{\mathbb{R}\}\}$
4. $\emptyset$
5. $\{\emptyset\}$
6. $\{\mathbb{R},\mathbb{Q},\mathbb{N}\}$
7. $\{\mathbb{R},\{\mathbb{Q},\mathbb{N}\}\}$
8. $\{\{0,1\},\{0,1,\{2\}\},\{0\}\}$

9. in general if |S| = n, what is |P(S)|?

H.

Write out the following sets by listing their elements between braces.

9. $\{X : X \subseteq \{3,2,a\} \text{ and } |X| = 2\}$
10. $\{X \subseteq \mathbb{N} : |X| \le 1\}$
11. $\{X : X \subseteq \{3,2,a\} \text{ and } |X| = 4\}$
12. $\{X : X \subseteq \{3,2,a\} \text{ and } |X| = 1\}$

Decide if the following statements are true or false. Explain.

13. $\mathbb{R}^3 \subseteq \mathbb{R}^3$
14. $\mathbb{R}^2 \subseteq \mathbb{R}^3$
15. $\{(x,y) : x - 1 = 0\} \subseteq \{(x,y) : x^2 - x = 0\}$
16. $\{(x,y) : x^2 - x = 0\} \subseteq \{(x,y) : x - 1 = 0\}$

I.

Find the indicated sets.

1. $\mathscr{P}(\{\{a,b\},\{c\}\})$
2. $\mathscr{P}(\{1,2,3,4\})$
3. $\mathscr{P}(\{\{\emptyset\},5\})$
4. $\mathscr{P}(\{\mathbb{R},\mathbb{Q}\})$
5. $\mathscr{P}(\mathscr{P}(\{2\}))$
6. $\mathscr{P}(\{1,2\}) \times \mathscr{P}(\{3\})$
7. $\mathscr{P}(\{a,b\}) \times \mathscr{P}(\{0,1\})$
8. $\mathscr{P}(\{1,2\} \times \{3\})$
9. $\mathscr{P}(\{a,b\} \times \{0\})$
10. $\{X \in \mathscr{P}(\{1,2,3\}) : |X| \le 1\}$
11. $\{X \subseteq \mathscr{P}(\{1,2,3\}) : |X| \le 1\}$
12. $\{X \in \mathscr{P}(\{1,2,3\}) : 2 \in X\}$

Suppose that $|A| = m$ and $|B| = n$. Find the following cardinalities.

13. $|\mathscr{P}(\mathscr{P}(\mathscr{P}(A)))|$
14. $|\mathscr{P}(\mathscr{P}(A))|$
15. $|\mathscr{P}(A \times B)|$
16. $|\mathscr{P}(A) \times \mathscr{P}(B)|$
17. $|\{X \in \mathscr{P}(A) : |X| \le 1\}|$
18. $|\mathscr{P}(A \times \mathscr{P}(B))|$
19. $|\mathscr{P}(\mathscr{P}(\mathscr{P}(A \times \emptyset)))|$
20. $|\{X \subseteq \mathscr{P}(A) : |X| \le 1\}|$

COURSE HOME PAGE DEPARTMENT HOME PAGE LOYOLA HOME PAGE