# Math 201: Class discussion 3 December 2019

#  Pigeon-Hole Principle



**I** State the basic pigeon-hole principle. State the strong pigeon-hole principle (*n* pigeon holes, and at least *kn+1* pigeons).

**II** Solve each of the following problems by using the pigeon-hole principle.

1. A bag contains balls of five colors: blue, purple, black, green and red. What is the *smallest* number of balls that must be drawn from the bag (without looking) so that among the drawn balls there are at least two of the same color? *(Who are the pigeons and what are the pigeon-holes?)*
2. Fifteen students in French 103 were given a dictation quiz. Albertine made 13 errors. Each of the other students made fewer errors. Prove that *at least two* students made the same number of errors.
3. There are 30 students in Spanish 103. On a dictation quiz, Carlos made 13 errors and all the other students made fewer errors. Prove that *at least three* students made the same number of errors.
4. Given 12 integers, show that two of them can be chosen whose difference is divisible by 11. (*Hint:* Let the pigeons represent the twelve integers, and let the numbers mod 11 be the pigeon holes.)
5. Show that among *n* positive integers there exist two whose difference is divisible by *n-1*.
6. Prove that for any *m* positive integers, the sum of some of these integers (possibly only one number) is divisible by *m*.
7. There are 50 people in a room. Some of them are acquainted with each other, some not. (Assume that “acquainted with” is a symmetric relation.) Prove that there are two persons in the room who have an equal number of acquaintances. (*Hint:* Let the pigeons be the 50 people. Consider two cases: Either everyone is acquainted with at least one other person, or else at least one person has no acquaintances.)
8. 51 points were placed, in an arbitrary way, into a unit square. Prove that one can find 3 of these points that are contained in a circle of radius 1/7. (*Hint:* Cut the square into 25 equal sub-squares.)



[Johann Peter Gustav Lejeune Dirichlet](http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Dirichlet.html) first stated the pigeon-hole

principle (also known as *Dirichlet’s box principle*) in 1834

