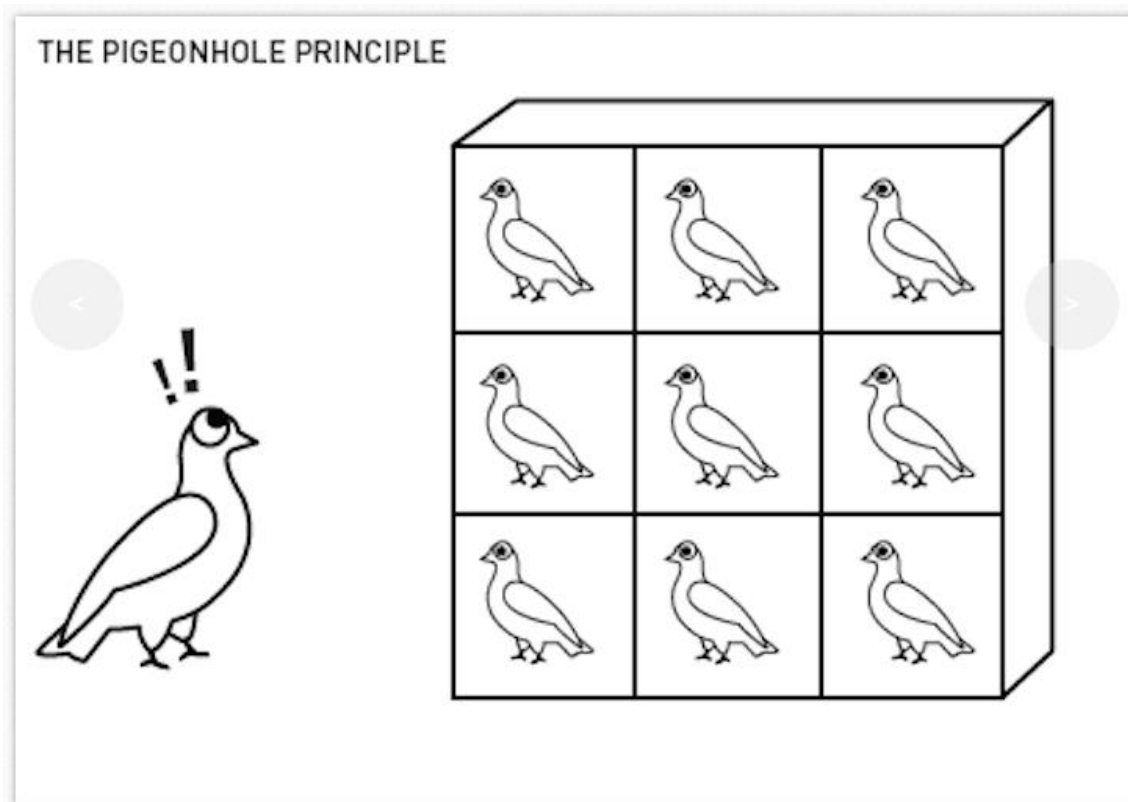


PIGEON-HOLE PRINCIPLE



- I** State the basic pigeon-hole principle. State the strong pigeon-hole principle (n pigeon holes, and at least $kn+1$ pigeons).
- II** Solve each of the following problems by using the pigeon-hole principle.
- A.** A bag contains balls of five colors: blue, purple, black, green and red. What is the *smallest* number of balls that must be drawn from the bag (without looking) so that among the drawn balls there are at least two of the same color? (*Who are the pigeons and what are the pigeon-holes?*)
 - B.** Fifteen students in French 103 were given a dictation quiz. Albertine made 13 errors. Each of the other students made fewer errors. Prove that *at least two* students made the same number of errors.
 - C.** There are 30 students in Spanish 103. On a dictation quiz, Carlos made 13 errors and all the other students made fewer errors. Prove that *at least three* students made the same number of errors.
 - D.** Given 12 integers, show that two of them can be chosen whose difference is divisible by 11. (*Hint: Let the pigeons represent the twelve integers, and let the numbers mod 11 be the pigeon holes.*)

- E.** Show that among n positive integers there exist two whose difference is divisible by $n-1$.
- F.** Prove that for any m positive integers, the sum of some of these integers (possibly only one number) is divisible by m .
- G.** There are 50 people in a room. Some of them are acquainted with each other, some not. (Assume that “acquainted with” is a symmetric relation.) Prove that there are two persons in the room who have an equal number of acquaintances. (*Hint:* Let the pigeons be the 50 people. Consider two cases: Either everyone is acquainted with at least one other person, or else at least one person has no acquaintances.)
- H.** 51 points were placed, in an arbitrary way, into a unit square. Prove that one can find 3 of these points that are contained in a circle of radius $1/7$. (*Hint:* Cut the square into 25 equal sub-squares.)



[Johann Peter Gustav Lejeune Dirichlet](#) first stated the pigeon-hole principle (also known as *Dirichlet's box principle*) in 1834

