**Math 201 Class Discussion 29 October 2019**

GCD and LCM

1. State the **well-ordering principle.**

Does it apply to the set of

(a) all positive real numbers?

(b) all non-negative rational numbers?

(c) all prime numbers?

(d) all integers?

(e) all positive irrational numbers?

2. State the **division algorithm**. Use the well-ordering principle to prove the division algorithm.

3. Show that gcd(n, m) = gcd(n + m, m).

4. Show that gcd(n, m) = gcd(n − m, m).

5. Give an example to show that gcd(n, m) = gcd(n + m, n − m) need not be true.

6. Suppose that n is even and gcd(n, m) = 5. Show that m is odd.

7. Find all numbers k with 0 ≤ k ≤ 100 such that gcd(100, k) = 5.

8. Find all numbers k with 0 ≤ k ≤ 100 such that gcd(100, k) = 4.

9. How many pairs of numbers (n, m), with 0 ≤ n, m ≤ 100 satisfy gcd(n, m) = 5.

10. How many pairs of numbers (n, m), with 0 ≤ n, m ≤ 100 satisfy gcd(n, m) = 8.

11. State the Euclidean Algorithm. Using the Euclidean Algorithm, find the

gcd(24, 15), gcd(60, 25), gcd(144, 100), gcd(101, 103),

gcd ( 18, 26), gcd(1071, 462), gcd(54321. 2021)

12. Define the LCM. Compute the LCM of

(a) 14 and 24 (b) 9 and 13, (c) 96 and 64

13. What is the relationship between GCD(p, q) and LCM(p, q)?

 **Part I:**

1. State the well-ordering principle.
2. Define gcd. Define *relatively prime.*
3. State and prove Euclid’s *division algorithm* (using the WOP).
4. Prove Euclid’s lemma.
5. State and prove the special case of the cancellation law.
6. State and prove Fermat’s Little Theorem

**Part II:**

1. Proof or counter-example: if $ac≡bc \left(mod n\right) then a≡b \left(mod n\right).$
2. There exist no integers a and b for which 21a + 30b = 1.
3. There exist no integers a and b for which 18a + 6b = 1
4. Use Fermat’s little theorem to show that 17 divides 11104+1.
5. If gcd(a, 35) = 1, show that a12 $≡1 \left(mod 35\right).$ Hint: Using Fermat, $a^{6}≡1 \left(mod 7\right) and $

$a^{4}≡1 \left(mod 5\right)$.

1. If gcd(a, 133) = gcd(b, 133) = 1, show that , for n ≥ 0, 133 is a divisor of a18 – b18.
2. If gcd(a, 42) = 1, prove that 163 = (3)(7)(8) divides a6 – 1.
3. Let a, b be integers. Then $a≡b \left(mod 6\right) if and only if a≡b \left(mod 2\right) and a≡b (mod 3)$
4. Find the units digit of 3100 by using Fermat.
5. Show that, for n ≥ 0, 13 is a divisor of 1112n+6 + 1.
6. The three most recent appearances of Haley’s comet were in the years 1835, 1910, and 1986. The next occurrence will be in 2061. Prove that

18351910 + 19862061 $≡0 \left(mod 7\right).$

1. Prove that a7 $≡a \left(mod 42\right)$ for all n.
2. Prove that a21 $≡a \left(mod 15\right)$ for all n.