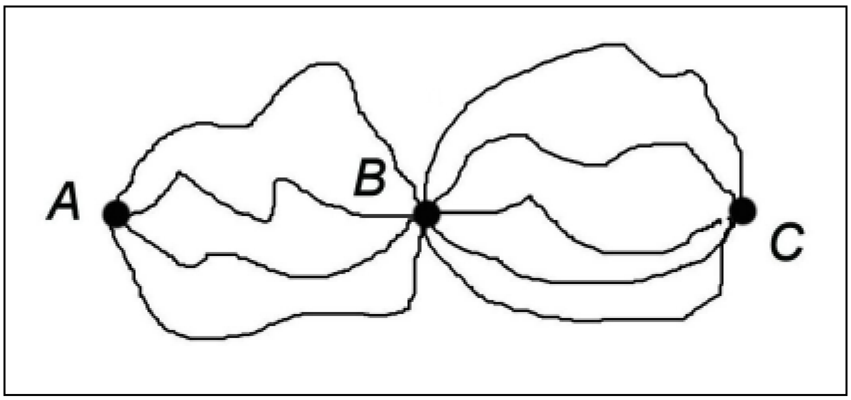
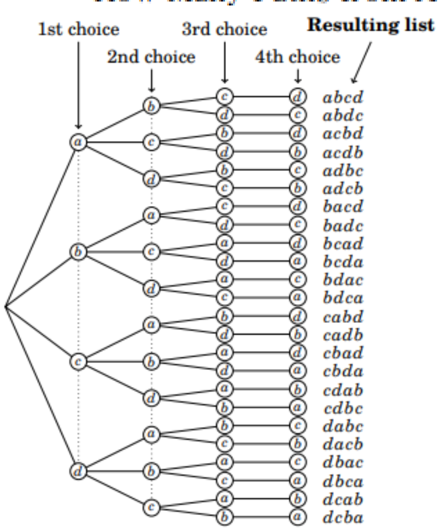
CLASS DISCUSSION: 17 SEPTEMBER 2019

COUNTING: AN INTRODUCTION

1. What is the multiplication principle? How many 3-character license plates can be manufactured if the first character is any upper-case letter, the second character is a vowel, and the third character is one-digit integer?



How Many Paths from A to C?



Constructing lists from letters in $\{a, b, c, d\}$, without repetition.

2. State the addition and subtraction principles.

3. (a) How many words of length 4 can you create using the letters a, b, c, and d?
   (b) What if no letter in the word can be repeated?
   (c) What if one letter is to appear exactly 3 times?
   (d) What if no letter can appear 4 times?

4. Same question as exercise 3, but this time we have four letters {a, b, c, d} in our alphabet.

Exercises for Section 3.1

Note: A calculator may be helpful for some of the exercises in this chapter. This is the only chapter for which a calculator may be helpful. (As for the exercises in the other chapters, a calculator makes them harder.)

1. Consider lists made from the letters T,H,E,O,R,Y, with repetition allowed.

(a) How many length-4 lists are there?

(b) How many length-4 lists are there that begin with T?

(c) How many length-4 lists are there that do not begin with T?

2. Airports are identified with 3-letter codes. For example, the Richmond, Virginia airport has the code *RIC*, and Portland, Oregon has *PDX*. How many different 3-letter codes are possible?

3. How many lists of length 3 can be made from the symbols A,B,C,D,E,F if...

(a) ... repetition is allowed.

(b) ... repetition is not allowed.

(c) ... repetition is not allowed and the list must contain the letter A.

(d) ... repetition is allowed and the list must contain the letter A.

4. Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such line-ups are there in which all 5 cards are of the same suit?

5. Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such line-ups are there in which all 5 cards are of the same color (i.e., all black or all red)?

6. Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such line-ups are there in which exactly one of the 5 cards is a queen?

7. This problem involves 8-digit binary strings such as 10011011 or 00001010 (i.e., 8-digit numbers composed of 0's and 1's).
   (a) How many such strings are there?
   (b) How many such strings end in 0?
   (c) How many such strings have the property that their second and fourth digits are 1's?
   (d) How many such strings have the property that their second **or** fourth digits are 1's?
8. This problem concerns lists made from the symbols A,B,C,D,E.
   (a) How many such length-5 lists have at least one letter repeated?
   (b) How many such length-6 lists have at least one letter repeated?
9. This problem concerns 4-letter codes made from the letters $A,B,C,D,\ldots,Z$.
   (a) How many such codes can be made?
   (b) How many such codes have no two consecutive letters the same?
10. This problem concerns lists made from the letters A,B,C,D,E,F,G,H,I,J.
   (a) How many length-5 lists can be made from these letters if repetition is not allowed and the list must begin with a vowel?
   (b) How many length-5 lists can be made from these letters if repetition is not allowed and the list must begin and end with a vowel?
   (c) How many length-5 lists can be made from these letters if repetition is not allowed and the list must contain exactly one A?
11. This problem concerns lists of length 6 made from the letters A,B,C,D,E,F,G,H. How many such lists are possible if repetition is not allowed and the list contains two consecutive vowels?
12. Consider the lists of length six made with the symbols P, R, O, F, S, where repetition is allowed. (For example, the following is such a list: (P,R,O,O,F,S).) How many such lists can be made if the list must end in an S and the symbol O is used more than once?

Exercises for Section 3.2

1. What is the smallest n for which $n!$ has more than 10 digits?

2. For which values of n does $n!$ have n or fewer digits?

3. How many 5-digit positive integers are there in which there are no repeated digits and all digits are odd?

4. Using only pencil and paper, find the value of $\frac{100!}{95!}$.

5. Using only pencil and paper, find the value of $\frac{120!}{118!}$.

6. There are two 0's at the end of $10! = 3,628,800$. Using only pencil and paper, determine how many 0's are at the end of the number 100!.

7. Compute how many 9-digit numbers can be made from the digits $1,2,3,4,5,6,7,8,9$ if repetition is not allowed and all the odd digits occur first (on the left) followed by all the even digits (i.e. as in 137598264, but not 123456789).

8. Compute how many 7-digit numbers can be made from the digits $1,2,3,4,5,6,7$ if there is no repetition and the odd digits must appear in an unbroken sequence. (Examples: 3571264 or 2413576 or 2467531, etc., but **not** 7234615.)

Exercises for Section 3.3

1. Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there that have at least one red card? How many such lineups are there in which the cards are either all black or all hearts?
2. Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there in which all 5 cards are of the same suit?
3. Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there in which all 5 cards are of the same color (i.e., all black or all red)?
4. Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such lineups are there in which exactly one of the 5 cards is a queen?
5. How many integers between 1 and 9999 have no repeated digits? How many have at least one repeated digit?
6. Consider lists made from the symbols A, B, C, D, E, with repetition allowed.
   - **(a)** How many such length-5 lists have at least one letter repeated?
   - **(b)** How many such length-6 lists have at least one letter repeated?
7. A password on a certain site must be five characters long, made from letters of the alphabet, and have at least one upper case letter. How many different passwords are there? What if there must be a mix of upper and lower case?
8. This problem concerns lists made from the letters $A, B, C, D, E, F, G, H, I, J$.
   - **(a)** How many length-5 lists can be made from these letters if repetition is not allowed and the list must begin with a vowel?
   - **(b)** How many length-5 lists can be made from these letters if repetition is not allowed and the list must begin and end with a vowel?
   - **(c)** How many length-5 lists can be made from these letters if repetition is not allowed and the list must contain exactly one A?
9. Consider lists of length 6 made from the letters A, B, C, D, E, F, G, H. How many such lists are possible if repetition is not allowed and the list contains two consecutive vowels?
10. Consider the lists of length six made with the symbols P, R, O, F, S, where repetition is allowed. (For example, the following is such a list: (P,R,O,O,F,S).) How many such lists can be made if the list must end in an S and the symbol O is used more than once?
11. How many integers between 1 and 1000 are divisible by 5? How many are not divisible by 5?

Combinatorics exercises: Stephan M. Wagner

1. How many possible ways are there to form five-letter words using only the letters A–H? How many such words consist of five distinct letters?

2. How many different number plates for cars can be made if each number plate contains two letters (A–Z) followed by five digits (0–9)?

3. We want to design a flag that consists of three horizontal stripes; the colour of the middle stripe should be different from the other two stripes. How many possibilities are there, if the colours red, green, blue, yellow, black and white can be used?

4. How many diagonals does a regular dodecagon (a twelve-sided polygon) have?

5. How many three-digit numbers abc have the property that $a \leq b \leq c$?

6. How many different four-digit numbers are there such that the product of the four digits is 420?

7. The dean of science wants to select a committee consisting of mathematicians and physicists to discuss a new curriculum. There are 15 mathematicians and 20 physicists at the faculty; how many possible committees of 8 members are there, if there must be more mathematicians than physicists (but at least one physicist) on the committee?

8. A *palindrome* is a word that can be read the same way in either direction (such as RACECAR). How many 9-letter palindromes (not necessarily meaningful) can be formed using the letters A–Z?

9. The four women Anne, Betsie, Charlotte and Dolores and the six men Eric, Frank, George, Harry, Ian and James are friends. Each of the women wants to marry one of the six men. In how many ways can this be done?

10. How many five-element subsets of $\{1, 2, 3, \ldots, 10\}$ contain at least one odd element?

How many really basic mathematical objects are there? One is surely the `miraculous' jar of the positive integers 1, 2, 3 . . . Another is the concept of a fair coin. Though gambling was rife in the ancient world and although prominent Greeks and Romans sacrificed to Tyche, the goddess of luck, her coin did not arrive on the mathematical scene until the Renaissance. Perhaps one of the things that had delayed this was a metaphysical position which held that God speaks to humans through the action of chance. . . . The modern theory begins with the expulsion of Tyche from the Pantheon. There emerges the vision of the fair coin, the biased coin. This coin exists in some mental universe and all modern writers on probability theory have access to it. They toss it regularly and they speculate about what they 'observe.'

- Philip Davis & Reuben Hersh, **The Mathematical Experience**