

**PRACTICE PROBLEMS (CHAPTERS 1 & 2)**

1. For each of the following strings, answer YES or NO, depending upon whether the string of symbols is a wff (well-formed formula) or not. You need not justify your answers.

(A)  $(p \rightarrow \sim q) \rightarrow (r \rightarrow (p \wedge q))$  \_\_\_\_\_

(B)  $\sim p$  \_\_\_\_\_

(C)  $pq \rightarrow \sim r$  \_\_\_\_\_

(D)  $p\sim$  \_\_\_\_\_

(E)  $(\sim p \wedge q) \rightarrow (r \vee (s \rightarrow t))$  \_\_\_\_\_

(F)  $p \wedge \rightarrow q$  \_\_\_\_\_

2. Find an *explicit* counterexample to the statement:

$$(A \cap B) \cup C = A \cap (B \cup C)$$

3. Using *truth tables*, determine if the following statements are logically equivalent or not.

$$p \rightarrow (q \wedge \sim q) \quad \text{and} \quad \sim p$$

4. Let  $U$  be the universe of all positive integers greater than 2.

Consider the following predicates:

$$P(x) = \text{“}x \text{ is a prime number”} \quad Q(x) = \text{“}x \text{ is odd”}$$

Express each of the following statements in symbolic form.

- (a) “ $x$  being prime is a *sufficient* condition for  $x$  being odd.”
- (b) “ $x$  being odd is a *necessary* condition for  $x$  being prime.”
- (c) “if  $x$  is odd and  $y$  is prime then  $x + y$  is not odd.”

5. Consider the universe of all people in the United States.

Define the following predicates:

$$P(x) = \text{“}x \text{ is a student”} \quad Q(x) = \text{“}x \text{ is smart”} \quad L(x, y) = \text{“}x \text{ loves } y\text{”}$$

Express each of the following in symbolic form.

- (a) All students are smart.
- (b) There exists a student.
- (c) There exists a smart student.
- (d) Every student loves some other student

- (e) No one loves anyone.
- (f) If Albertine loves anyone, then she loves Swann.
- (g) There is a student who loves himself but who is loved by *no one else*.

**6.** Negate each of the following sentences. Your answer should be a sentence in English – not a symbolic sentence. (Nor should you simply write: “It is not true that ....”).

- (a) You can fool all of the people all of the time.
- (b) If  $\cos x = 7$  then it is not true that  $x$  is an integer.
- (c) If the cat ate the mouse or the mouse ate the flea, then Albertine will either watch Dark on Netflix or go swimming in the lake.
- (d) You cannot teach a cat to fetch, but you can teach a dog to swim.

**7.**

State the negation of the following quantified propositions.

- a)  $(\forall x \in S)(x \geq 0)$
- b)  $(\exists n \in S)(n \text{ is a prime number})$
- c)  $(\forall x)(\exists y)(xy = 10)$  (which we read “for all  $x$ , there exists a  $y$  such that  $xy = 10$ ”)
- d)  $(\exists x)(\forall y)(xy \neq 10)$  (which we read “there exists an  $x$  such that for all  $y$  we have  $xy \neq 10$ ”)

**8.** Is the statement  $(p \vee \sim q) \vee (\sim p \wedge q)$  a tautology?

The following are Florida State University questions:

**9.**

Use DeMorgan's Laws to write the negation of each statement:

- (a) 1. I want a car and a motorcycle. (b) My cat stays outside, or it makes a mess. (c) I've fallen, and I can't get up.
- (d) You study, or you do not get a good grade.

**10.** Select the statement that is the negation of "Today is Monday, and it isn't raining."

- A. Today isn't Monday, and it isn't raining.
- B. Today isn't Monday, or it isn't raining.
- C. Today isn't Monday, or it is raining.

- D. Today isn't Monday, and it is raining.
- E. Today is Friday, and it is snowing.

**11.** Select the statement that is the negation of "I'm careful, or I make mistakes."

- A. I'm not careful, and I don't make mistakes.
- B. I'm not careful, or I don't make mistakes.
- C. I'm not careful, and I make mistakes.
- D. I'm not careful, or I make mistakes.
- E. I never make mistakes. *[sic]*

**12.** Select the statement that is the negation of "I walk, or I chew gum."

- A. I don't walk, and I chew gum.
- B. I don't walk, or I chew gum.
- C. I don't walk, and I don't chew gum.
- D. I don't walk, or I don't chew gum.
- E. I walk until I step on chewed gum.

**13.** Select the statement that is the negation of "I'm mad as heck, and I'm not going to take it anymore."

- A. I'm not mad as heck, and I'm not going to take it anymore.
- B. I'm not mad as heck or I'm not going to take it anymore.
- C. I'm not mad as heck, and I am going to take it anymore.
- D. I'm not mad as heck, or I am going to take it anymore.

**14.** Select the statement that is the negation of "All of the businesses are closed."

- A. Some of the businesses are closed.
- B. Some of the businesses are not closed.
- C. None of the businesses are closed.
- D. All of the businesses are open.
- E. All of my clothes are businesslike.

**15.** Select the statement that is the negation of the statement "Some pilots are pirates."

- A. All pilots are pirates.
- B. No pilots are pirates.
- C. Some pilots are not pirates.
- D. All pirates are pilots.

**16.** *'But wait a bit,' the Oysters cried, 'Before we have our chat;  
For some of us are out of breath, And all of us are fat!'  
'No hurry!' said the Carpenter. They thanked him much  
for that.*

Select the statement that is the negation of "Some of us are out of breath, and all of us are fat."

- A. Some of us aren't out of breath, or none of us is fat.
- B. Some of us aren't out of breath, and none of us is fat.

- C. None of us is out of breath, and some of us aren't fat.
- D. None of us is out of breath, or some of us aren't fat.

**17.** Select the statement that is the negation of "All summer days are muggy."

- A. All muggy days are summer.
- B. Some summer days are muggy.
- C. Some summer days are not muggy.
- D. No summer days are muggy.

**18.** Select the statement that is the negation of "Some weasels are cuddly."

- A. No weasels are cuddly.
- B. All weasels are cuddly.
- C. Some weasels are not cuddly.
- D. All cuttlefish are weasely.

**19.** Select the statement that is the negation of "Coach Spurrier is charming, and Coach Spurrier is modest."

- A. Coach Spurrier is not charming, and Coach Spurrier is not modest.
- B. Coach Spurrier is not charming, or Coach Spurrier is not modest.
- C. Coach Spurrier is not charming, and Coach Spurrier is modest.
- D. Let's get serious for a minute.

**20.** Select the statement that is the negation of "The speed limit is 55 mph, and granny is driving 35 mph."

- A. The speed limit is not 55 or granny is not driving 35.
- B. The speed limit is not 55 and granny is not driving 35
- C. The speed limit is not 55 or granny is driving 35.
- D. The speed limit is not 55 and granny is driving 35.
- E. Counseling for road rage is available at 1 - 900 - calmdown