**MATH 201 Practice Problems for Test III**

(Revised)



1. For any real numbers, *c* and *d*, let us define the binary operation  as follows:

c  d = c2 + d2 – 1

Give either a *brief* justification or counterexample for each of the following assertions:

1. The set of integers is closed under the operation  .
2. The set of even integers is closed under the operation  ?
3. The set of odd integers is closed under .
4. The set of positive integers is closed under .
5. The set of rational numbers is closed under .
6. The set of irrational numbers is closed under .
7. Let S be the set of all polynomials for which odd powers of x (i.e., 1, 3, 5, 7,…) do not appear.

For example, 4 + ½ x2 –  x8 .

(a) Is S closed under the operation of differentiation? Why?

1. Is S closed under the operation of taking the second derivative?
2. Is S closed under the operation multiplication by x3?
3. Is S closed under the operation multiplication by x4?

(e) Is S closed under the following unary operation, ● ?

 For all p

Note: that polynomial p is evaluated at 1 + x4

(f) Is S closed under the following binary operation ☹?

 For all p Here as before, we interpret p(1 + x) is the polynomial p evaluated at 1 + x.

1. Define the following binary relation R on **N**. For c, d ∈ **Z+**, cRd if |c – d | < 5. *(Justify each answer!)*

 (a) Is R reflexive? Why?

 (b) Is R symmetric? Why?

 (c) Is R transitive? Why?

1. Define the binary relation *R* on a non-empty set, S, of books as follows:

 For a, b ∈ S, a*R*b if book ***a*** costs more **and** contains fewer pages than book ***b***. *(Justify each answer!)*

 (a) Is R reflexive? Why?

(b) Is R symmetric? Why?

 (c) Is R transitive? Why?

1. Again, let S be a non-empty set of books. Let *H* be the binary relation defined by:

 for *a*, *b* ∈ S, a*H*b if book ***a*** costs more ***or*** contains fewer pages than book ***b***. *(Justify each answer!)*

(a) Is *H* reflexive? Why?

 (b) Is *H* symmetric? Why?

 (c) Is *H* transitive? Why?

1. Let *x* and *y* be real numbers. Prove that if x2 + 5y = y2 + 5x. Prove that either x = y or x + y = 5.
2. Using the *Euclidean algorithm*, find the gcd of 7701543 and 3141
3. Using the method of contrapositive proof, prove each of the following statements:
4. Suppose that x, y Z. If x2(y + 3) is even, prove that x is even or y is odd.
5. Let a Z. Prove that if a2 is not divisible by 4, then a is odd.
6. Prove that there exist no integers a and b for which 390a + 63b = 1.
7. Use Fermat’s little theorem to show that 17 divides 11104+1.
8. Prove that is irrational.
9. Let a Z. Prove that
10. Let a, b **Z**. Prove that (a – 3)b2 is even if and only if a is odd and b is even.
11. The three most recent appearances of Haley’s comet were in the years 1835, 1910, and 1986.

The next occurrence will be in 2061. Prove that 18351910 + 19862061

1. Find 201313 (mod 13)
2. Let W be the set of all words in the 2019 edition of the Oxford English dictionary. Define xRy if x and y have at least one letter in common. Is R reflexive? Symmetric? Transitive?
3. Let A= R3. Let a∼b mean that a and b have the same z coordinate. Does this define an equivalence relation?
4. Let S be a finite set and A=P(S), the power set of S. For any a,b∈A, let a∼b mean that a and b have the same cardinality (that is the same number of members).

Show that ∼ is an equivalence relation. Compute the equivalence classes when S ={1,2,3}.

1. The following purports to prove that the reflexivity condition is unnecessary, that is, it can be derived from symmetry

and transitivity:

Suppose a∼b. By symmetry, b∼a. Since a∼b and b∼a, by transitivity, a∼a. Therefore, ∼ is reflexive.

What's wrong with this argument?

1. True or False? Explain.
2. If *R* is an equivalence relation on a finite non-empty set A, then the equivalence classes of *R* all have the same number of elements.
3. Let *R* be a relation defined on the set **Z** by aRb if a ≠ b. Then R is symmetric and transitive.
4. Define GCD. Prove that gcd(n, m) = gcd(n + m, m).
5. Show that gcd(n, m) = gcd(n − m, m).
6. Give an example to show that gcd(n, m) = gcd(n + m, n − m) need not be true.
7. Suppose that n is even and gcd(n, m) = 5. Show that m is odd.
8. Find all numbers k with 0 ≤ k ≤ 100 such that gcd(100, k) = 5.
9. (a) Show that there is no integer x satisfying the equation 2x + 1 = 5x – 4

(b) Show that there is no integer x satisfying the equation 18x2 + 39x – 7 = 0

(c) Show that the system of equations

11x – 5y = 7

9x + 10y = -3

has no integer solution.

1. Let A = {1, 2, 3, 4, 5}, and let

*R* = {(1, 1), (1, 3), (1, 4), (2, 2), (2, 5), (3, 1), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4), (5, 2), (5, 5)} define an equivalence relation on A. (You may wish to check this!) Which of the following is an *equivalence class*?

1. {1, 2, 3} b) {2, 3, 5} c) {1, 3, 4} d) {1, 2} e) {1, 2, 3, 4, 5}
2. State Euclid’s lemma. Prove that gcd(a, bc) = 1 if and only if gcd(a, b) = 1 and gcd(a, c) = 1.
3. Let X denote the set of all continuous functions on [0, 1].
4. Is X closed under addition, subtraction, multiplication, division?
5. Is X closed under composition? Differentiation?
6. Let A be the set of all polynomials in the variable x. Is A closed under:
7. Addition, subtraction, multiplication, composition, differentiation?
8. Consider the unary operations in A defined by, for

 (ii) U(p) = (iii) U(p) = x2p(x)

Under which of these operations is A closed?

1. (a) Prove that is irrational.

(b) Find 22019 mod 17

1. Find 22019 mod 33.
2. Find the units and ten digits of (984321)(454443)(444555).
3. Let Y be the power set of a finite non-empty set F.
4. Is Y closed under complements?
5. Is Y closed under union? Intersection?