MATH 201 SOLUTIONS: QUIZ I

12 SEPTEMBER 2019

1. *[10 pts]* Negate the following statement. (Your answer should be a complete sentence written in English.)

If all rich people are happy, then all poor people are sad.

Solution: The negation of $p \Rightarrow q$ *is* $p \wedge (\sim q)$.

Hence the negation of the given statement is:

All rich people are happy and at least one poor person is not sad.

2. *[15 pts]* Let p = "Aldo is Italian" and q = "Frederick is British."

Write each of the following as a statement in propositional logic.

(a) Aldo is not Italian.

$\sim p$

(b) Aldo is Italian, or if Aldo is not Italian, then Frederick is British.

$p \vee (\sim p \Rightarrow q)$

(c) Either Aldo is Italian and Frederick is British, or neither Aldo is Italian nor Frederick is British.

$(p \wedge q) \vee (\sim p \wedge \sim q)$

3. *[10 pts]* Negate the following sentence: $\forall a \in X \ \exists b \in Y \ \ 13 > ab > 5$

$$\exists a \in X \ \forall b \in Y \ \ (ab \leq 5) \vee (ab \geq 13)$$

4. *[15 pts]* Use a truth table to determine if the following statement is always true:

$$(\sim p) \Rightarrow \big((p \wedge \sim q) \Rightarrow (p \wedge q)\big)$$

Yes, the statement is a always true.

"Todd, trust math. As in Matics, Math E. First-order predicate logic. Never fail you. Quantities and their relation. Rates of change. The vital statistics of God or equivalent. When all else fails. When the boulder's slid all the way back to the bottom. When the headless are blaming. When you do not know your way about. You can fall back and regroup around math. Whose truth is deductive truth. Independent of sense or emotionality. The syllogism. The identity. Modus Tollens. Transitivity. Heaven's theme song. The night light on life's dark wall, late at night. Heaven's recipe book. The hydrogen spiral. The methane, ammonia, H_2O. Nucleic acids. A and G, T and C. The creeping inevitability. Caius is mortal. Math is not mortal. What it is is: listen: it's true."

– David Foster Wallace, **Infinite Jest**