MATH 201 Solutions: quiz II 19 September 2019

*In each of the following questions, be certain to justify your answers!*

# *[16 pts.]* How many integers between 10,000 and 100,000 have no digits other than

# 6, 7, or 8?

**Solution:** Here, we count all the integers between 66,666 and 88,888, inclusive.

Since the first digit is one of three characters {6, 7, 8} and similarly for each of the remaining five digits, the total number of such integers is **35**.

1. 6, 7, 8, or 0?

**Solution:** Once again, we must count the integers between 66,666 and 88,888, but this time the first digit cannot be 0. The total number of such integers is $3∙4^{4}.$

# *[16 pts.]* Vladimir flips a quarter 10 times in a row and records the 10 outcomes (H or T for each flip).

#  How many possible sequences are there?

**Solution:**

We are dealing with a sequence of 10 characters; each character can be H or T.

The number of such sequences is **210**.

1. How many sequences contain only 1 head?

**Solution:** The one head may be placed in any one of 10 slots. Once that special slot is chosen, the other must all be T. Thus, the number of possibilities is **10**.

1. How many sequences contain a match on the first and last flips (that is, either a head on both first and last flips or a tail on both first and last flips)

**Solution:**

We can fill the slots from the second through the ninth with any of 28 sequences of H, T.

Now we have but two choices for first and last place: Either two Hs or two Ts.

Thus, the number of such 10-character sequences is $2∙2^{8}=2^{9}.$

1. *(extra credit)* How many sequences contain 7 heads and a run of 3 tails? For example, HHHTTTHHHH.

**Solution:**

Consider the sequence of 7 heads; the three tails constitute a single unit and as such can be placed before anyone of the 7 heads or after the last head. Thus, there are **8** such strings.

1. *[16 pts.]* In how many ways can you draw 3 cards from a standard deck of 52, where *order matters*, in such a way that
2. you have exactly two Jacks?

**Solution:**

There are 3 positions in which the “non-Jack” may appear (first, second, third card). Here we choose one of the 3 slots. The slot may be filled with any one of 48 cards.

To fill the first empty position there are 4 Jacks from which to choose. Finally, the remaining slot can be filled with any of the 3 remaining Jacks.

Hence, the total number of ways one can draw three cards subject to the given rules is

$ 3∙ 4∙3∙48. $

1. you have three different suits represented?

**Solution:**

The first card may be chosen independently of rank or suit, thus any of 52.

The second card must be of a new suit, hence 39 possibilities.

Finally, the third card must be of one of the remaining two suits, hence 26 options.

Therefore, the total number of such sequences is $52∙39∙26. $

1. each card is either a heart or a spade?

**Solution:**

There are 26 hearts and spades in the deck of 52. Thus, the first card may be any one of 26 cards; the second card must be any one of the remaining 25 cards; and the third card any one of 24.

Hence the total number of such sequences is $26∙25∙24. $

