MATH 201 Solutions: quiz II 19 September 2019

*In each of the following questions, be certain to justify your answers!*

# *[16 pts.]* How many integers between 10,000 and 100,000 have no digits other than

# 6, 7, or 8?

**Solution:** Here, we count all the integers between 66,666 and 88,888, inclusive.

Since the first digit is one of three characters {6, 7, 8} and similarly for each of the remaining five digits, the total number of such integers is **35**.

1. 6, 7, 8, or 0?

**Solution:** Once again, we must count the integers between 66,666 and 88,888, but this time the first digit cannot be 0. The total number of such integers is

# *[16 pts.]* Vladimir flips a quarter 10 times in a row and records the 10 outcomes (H or T for each flip).

# How many possible sequences are there?

**Solution:**

We are dealing with a sequence of 10 characters; each character can be H or T.

The number of such sequences is **210**.

1. How many sequences contain only 1 head?

**Solution:** The one head may be placed in any one of 10 slots. Once that special slot is chosen, the other must all be T. Thus, the number of possibilities is **10**.

1. How many sequences contain a match on the first and last flips (that is, either a head on both first and last flips or a tail on both first and last flips)

**Solution:**

We can fill the slots from the second through the ninth with any of 28 sequences of H, T.

Now we have but two choices for first and last place: Either two Hs or two Ts.

Thus, the number of such 10-character sequences is

1. *(extra credit)* How many sequences contain 7 heads and a run of 3 tails? For example, HHHTTTHHHH.

**Solution:**

Consider the sequence of 7 heads; the three tails constitute a single unit and as such can be placed before anyone of the 7 heads or after the last head. Thus, there are **8** such strings.

1. *[16 pts.]* In how many ways can you draw 3 cards from a standard deck of 52, where *order matters*, in such a way that
2. you have exactly two Jacks?

**Solution:**

There are 3 positions in which the “non-Jack” may appear (first, second, third card). Here we choose one of the 3 slots. The slot may be filled with any one of 48 cards.

To fill the first empty position there are 4 Jacks from which to choose. Finally, the remaining slot can be filled with any of the 3 remaining Jacks.

Hence, the total number of ways one can draw three cards subject to the given rules is

1. you have three different suits represented?

**Solution:**

The first card may be chosen independently of rank or suit, thus any of 52.

The second card must be of a new suit, hence 39 possibilities.

Finally, the third card must be of one of the remaining two suits, hence 26 options.

Therefore, the total number of such sequences is

1. each card is either a heart or a spade?

**Solution:**

There are 26 hearts and spades in the deck of 52. Thus, the first card may be any one of 26 cards; the second card must be any one of the remaining 25 cards; and the third card any one of 24.

Hence the total number of such sequences is

