MATH 201 SOLUTIONS: QUIZ II 19 SEPTEMBER 2019

In each of the following questions, be certain to justify your answers!

1. *[16 pts.]* How many integers between 10,000 and 100,000 have no digits other than

(a) 6, 7, or 8?

Solution: Here, we count all the integers between 66,666 and 88,888, inclusive.
Since the first digit is one of three characters {6, 7, 8} and similarly for each of the remaining five digits, the total number of such integers is $\mathbf{3^5}$.

(b) 6, 7, 8, or 0?

Solution: Once again, we must count the integers between 66,666 and 88,888, but this time the first digit cannot be 0. The total number of such integers is $\mathbf{3 \cdot 4^4}$.

2. *[16 pts.]* Vladimir flips a quarter 10 times in a row and records the 10 outcomes (H or T for each flip).

(a) How many possible sequences are there?

Solution:
We are dealing with a sequence of 10 characters; each character can be H or T.
The number of such sequences is $\mathbf{2^{10}}$.

(b) How many sequences contain only 1 head?

Solution: The one head may be placed in any one of 10 slots. Once that special slot is chosen, the other must all be T. Thus, the number of possibilities is **10**.

(c) How many sequences contain a match on the first and last flips (that is, either a head on both first and last flips or a tail on both first and last flips)

Solution:
We can fill the slots from the second through the ninth with any of 2^8 sequences of H, T.
Now we have but two choices for first and last place: Either two Hs or two Ts.
Thus, the number of such 10-character sequences is $2 \cdot 2^8 = \mathbf{2^9}$.

(d) (extra credit) How many sequences contain 7 heads and a run of 3 tails? For example, HHHTTTHHHH.

Solution:

Consider the sequence of 7 heads; the three tails constitute a single unit and as such can be placed before anyone of the 7 heads or after the last head. Thus, there are **8** such strings.

3. *[16 pts.]* In how many ways can you draw 3 cards from a standard deck of 52, where *order matters*, in such a way that

(a) you have exactly two Jacks?

Solution:

There are 3 positions in which the "non-Jack" may appear (first, second, third card). Here we choose one of the 3 slots. The slot may be filled with any one of 48 cards.
To fill the first empty position there are 4 Jacks from which to choose. Finally, the remaining slot can be filled with any of the 3 remaining Jacks.
Hence, the total number of ways one can draw three cards subject to the given rules is $\mathbf{3 \cdot 4 \cdot 3 \cdot 48}$.

(b) you have three different suits represented?

Solution:

The first card may be chosen independently of rank or suit, thus any of 52.
The second card must be of a new suit, hence 39 possibilities.
Finally, the third card must be of one of the remaining two suits, hence 26 options.
Therefore, the total number of such sequences is $\mathbf{52 \cdot 39 \cdot 26}$.

(c) each card is either a heart or a spade?

Solution:

There are 26 hearts and spades in the deck of 52. Thus, the first card may be any one of 26 cards; the second card must be any one of the remaining 25 cards; and the third card any one of 24.
Hence the total number of such sequences is $\mathbf{26 \cdot 25 \cdot 24}$.

"Of course we're not going to experiment on you - we just needed another hand for our bridge game!"