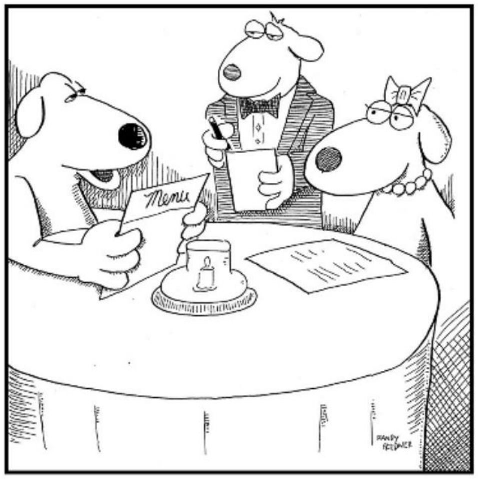
HOMEWORK: MATH 201



"I'll have the math homework."

Homework 0: Due: Friday, 18th January

Briefly relate (in one or two paragraphs) information about yourself that will help me get to know you. If you wish, you may let the following questions serve as a guide: Which other courses in math have you taken or are taking concurrently with Math 201. Why have you chosen to take Math 201 now? (for example: *"major requirement", "minor requirement", "just for fun because I love mathematics", "nothing else fits my schedule", "my parents forced me to take this course", "I am looking for an easy A to raise my GPA"*); what is your major?; what is your career goal?; what has been the nature of your previous experience with math either in high school or in college (that is, have you enjoyed math in the past?).
(Please post your response as a private message in Piazza no later than midnight, Friday. For "Subject" write "**201 Homework 0**") Thank you.

Homework 1: Due: Wednesday, 23rd January

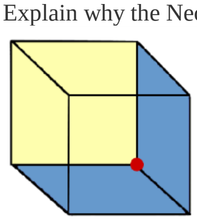
Read sections 1.1 through 1.7 of Hammack. Note that all of Hammack's exercises have answers or solutions at the end of the book. So you would profit by doing many of the exercises in these sections and, if you wish, check your answers.

(1) Answer either (1a) *OR* (1b) (I hope you find this to be an enjoyable exercise.)

(**1a**) Watch the famous Abbott and Costello video at https://www.youtube.com/watch?v=kTcRRaXV-fg
Write an analysis of whether the language of the video makes any sense.
Either precisely explain why the statements are logical or explain why this routine is nonsense.
Be certain that you are clear and unambiguous in what you write.

(**1b**) Watch the Colbert video, https://www.youtube.com/watch?v=JgMiS81jFyE
Is this nonsense, or can you instill some logic to this dialogue?

(2) Explain why the Necker cube is an example of visual ambiguity:



(3) Explain why each of the following sentences is ambiguous.

- **a.** Assume you wish to increase your wealth, would you rather be paid $400 weekly OR $400 biweekly?
- **b.** Did you see her duck?
- **c.** This is a good sign!
- **d.** We don't just serve hamburgers; we serve people.
- **e.** Slow children at play.
- **f.** Automatic washing machines. Please remove all your clothes when the light goes out.
- **g.** Please wait for the hostess to be seated.

(4) Let X = {0, 1, 2, 3, 4, 5, 6}

- **a.** Find |X|
- **b.** Define a function S on P(X) as follows:
  For $A \in P(X)$, let $S(A)$ be the sum of all the elements of A.
  For example $S(\{3, 5, 6\}) = 14$.
  Define Y = {A∈ $P(X)$| S(A) = 5}.
  List all the elements of Y. Find |Y|.
- **c.** Let A = {4, {0}, {1, 3}}, B = {{1, 2}, 3, 4, {3, 4}, {0}} and C = {{1, 3}, {0, 1, 5}, 3, {4}, {0}, 4}.

Find |B|, |C|, |A∪B|, |A∩B|, |A – B|, by first listing the elements of each set.

(5) Let A, B, and C be subsets of the set S. Using only the operators for the union, intersection, difference, and complement as well as the letters A, B, and C, write down expressions for each of the following.

(Note that "event" is simply another way of referring to a subset.) *(Also, answers are not unique.)*

- **a.** at least one event is true
- **b.** only event A is true
- **c.** A and B are true, but C is not
- **d.** all events are true
- **e.** none of the events is true
- **f.** *exactly* one event is true
- **g.** *at most* two events are true
- **h.** *exactly* two events are true

Bertrand Russell

Homework 2: Due: Monday, 28th January

Review chapter 1. Begin reading the first few sections of chapter 2.

1. Let X = {p, q}. List all the elements of P(P(X)).

2. Prove that, for any universe, and any sets A and B,

$$If\ A \subseteq B\ then\ \bar{B} \subseteq \bar{A}$$

Use complete sentences.

3. Prove that, for any universe, and any sets A, B, and C,

$$C - (A \cap B) = (C - A) \cup (C - B)$$

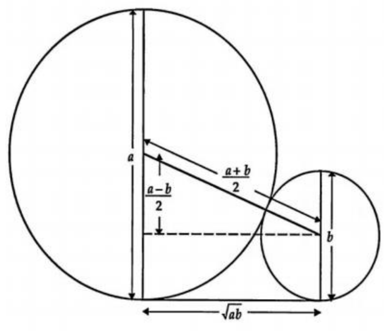
Remember our method of proving the equality of two sets!

Use complete sentences.

4. *Proof without words:* Using the clever picture below, give a precise and clear explanation of the

Arithmetic Mean-Geometric Mean Inequality

$\frac{a+b}{2} \geq \sqrt{ab}$ with **equality** iff a = b



5. **(Extra credit):**

Prove that, for any universe, and any sets A, B, and C, in the universe

$$C - (B - A) = (A \cap C) \cup (C - B)$$

Use complete sentences.

Homework 3 (revised, Jan 29th): Due: Wednesday, 6th February

Read sections 2.1 through 2.5 of Hammack.

1. (a) In propositional logic, ***modus ponendo ponens*** (Latin for "the way that affirms by affirming"; generally abbreviated to **MP** or ***modus ponens***) or ***implication elimination*** is a rule of inference. It can be summarized as "*p* implies *q* and *p* is asserted to be true, so therefore *q* must be true," viz, $(p \wedge (p \Rightarrow q)) \Rightarrow q$. The history of *modus ponens* goes back to antiquity. Using a truth table, prove *modus ponens*.

(b) Consider the two sentences $\mathcal{A}$ and $\mathcal{B}$ defined by:

$\mathcal{A}$: $(p \wedge q) \Rightarrow r$

$\mathcal{B}$: $p \Rightarrow (q \Rightarrow r)$

Does $\mathcal{A} \Rightarrow \mathcal{B}$?

Does $\mathcal{B} \Rightarrow \mathcal{A}$?

(Of course, use truth tables to answer these questions.)

(c) Negate each of the following sentences:

(i) $a \Rightarrow b \wedge c$

(ii) $(a \wedge b) \vee (a \wedge b)$

(iii) $(a \wedge b) \Leftrightarrow (\sim a \vee \sim b)$

(iv) $(a \Rightarrow b) \Rightarrow (\sim c \Rightarrow (b \Rightarrow a)$

2. Let's consider a propositional language where

- A = "Albertine comes to the party,"
- B = "Boris comes to the party,"
- C = "Cordelia comes to the party,"
- D = "Dmitri comes to the party."

Formalize each of the following sentences:

a. "If Dmitri comes to the party then Boris and Cordelia come too."
b. "Cordelia comes to the party only if Albertine and Boris do not come."
c. "Dmitri comes to the party if and only if Cordelia comes and Albertine doesn't come."

d. "If Dmitri comes to the party, then, if Cordelia doesn't come then Albertine comes."
e. "Cordelia comes to the party provided that Dmitri doesn't come.
f. "A necessary condition for Albertine coming to the party is that, if Boris and Cordelia aren't coming, Dmitri comes."
g. "Albertine, Boris and Cordelia come to the party if and only if Dmitri doesn't come, but, if neither Albertine nor Boris come, then Dmitri comes only if Cordelia comes "but, if Dmitri comes, then Boris doesn't come"

3. *Express each of the following statements in predicate logic. Define your "atomic predicate symbols"; also, give the domain of every variable that you use. Assume that the "or" in a sentence is non-restrictive.*

(a) Nobody in Math 201 is smarter than everybody in Math 162.

(b) Everyone likes Albertine except Albertine herself.

(c) If Odette can do the task, anyone can.

4. (A) Decide whether each of the following statements is true or false, where $x, y, z \in \mathbb{Z}$. Give proof or counterexample. If false, then write the negation of the sentence.

(a) $\forall x\, \exists y\, (2x - y = 0)$

(b) $\exists y\, \forall x\, (x - 2y = 0)$

(c) $\forall x\, \exists y (x - 2y = 0)$

(d) $\forall x\;\; x < 10\;\; \Rightarrow \forall y\, (y < x \Rightarrow y < 9)$

(e) $\exists y\, \exists z\;\; y + z = 100$

(f) $\forall x\, \exists y\;\; (y > x\; \wedge\; \exists z\;\; y + z = 100)$

(B) Repeat part (A) now assuming that x, y, z ∈ ℝ.

Homework 4 Due: Wednesday, 13th February (*Recommended:* solve these exercises before the test on Feb 8th)

Review sections 2.1 through 2.5 of Hammack. Read carefully sections 2.6 through 2.12. Pay particular attention to section 2.10.

The majority of the following exercises from the **Book of Proof** require little time. They offer a chance to review chapter 2 in preparation for Test 1 on Feb. 8.

Solve: Section 2.3/ 2, 4, 8, 10, 12; section 2.4/ 2, 4; section 2.5/ 2, 10; section 2.6/ 10; section 2.7/ 2, 4, 8, 10; and section 2.9/2, 4, 8, 12

If time permits, you may wish to begin reading chapter 3.

Homework 5 Due: Wednesday, 20th February

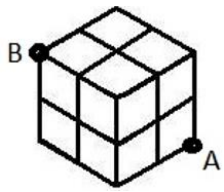
Read carefully sections 3.1 through 3.5 and 3.8 of Hammack.

I How many non-negative integer solutions are there to the equation: $x_1 + x_2 + x_3 + x_4 + x_5 = 99$?

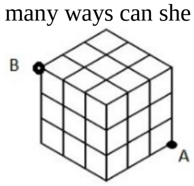
(b) Same question as (a), but now assume that the solution must consist of *positive* integers.

(c) Same question as (a) except *at least one of the components* of a solution $(x_1, x_2, x_3, x_4, x_5)$ must be 0. For example, $97 + 1 + 0 + 0 + 1 = 99$ is one such solution.

II (a) Consider a $2 \times 2 \times 2$ cube, as illustrated below. Charlotte, a spider, wants to travel from A to B; she can only walk on the lines. The path must be the shortest (i.e., 2 up, 2 left, and 2 forward). In how many ways can Charlotte travel?



(b) Developing increased self-confidence, Charlotte now wishes to travel on a 3 × 3 × 3 cube subject to the same conditions as in part (a). In how many ways can she travel?



III (Hammack exercises)

Section 3.3/ exercises 4, 8, 10, 12

Section 3.4/ exercises 8, 10, 16

Section 3.5/ exercises 4, 12, 18

Section 3.8/ exercises 4, 8, 10, 16

Homework 6 Due: Wednesday, 27th February

Read carefully sections 10.1 and 10.2 of Hammack.

Solve exercises for chapter 10 (pg 195-916)

6, 8, 10, 12, 16, 18, 20, 34

Homework 7: Due: Wednesday, 13th March

Study chapters 4, 5, and 6 of Hammack. Solve the following exercises.
126/ exercise 6, 10, 14;

136/ exercise, 8, 12, 18, 20, 24;

144/ exercise 2, 4, 6, 10, 18, 24

Test II: March 15th, Beware the Ides of March

Homework 8: Due: Wednesday, 20 March

Study carefully chapter 7. Also, study the "well-ordering principle," the proof of the Division Algorithm (pp. 30 – 31), the definition of gcd, and the proof of Fermat's little theorem.

Review Euclid's proof of the existence of infinitely many prime numbers.

Using Fermat's little theorem, compute 3^{31} (mod 7), 29^{25} (mod 11), and 128^{129} (mod 17).

Solve 155/ 8, 12, 16, 24, 28, 30, 34

Homework 9: Due: Monday, 1 April

Study carefully sections 11.1, 11.2, and 11.3.

Solve: 11.1/ exercises 10, 12, 14; 11.2/ exercises 14, 18; 11.3/ exercises 6, 8, 12, 16

Homework 10: Due: Monday, 8 April

Read carefully chapter 12.

A summary of basic concepts:

https://brilliant.org/wiki/bijection-injection-and-surjection/

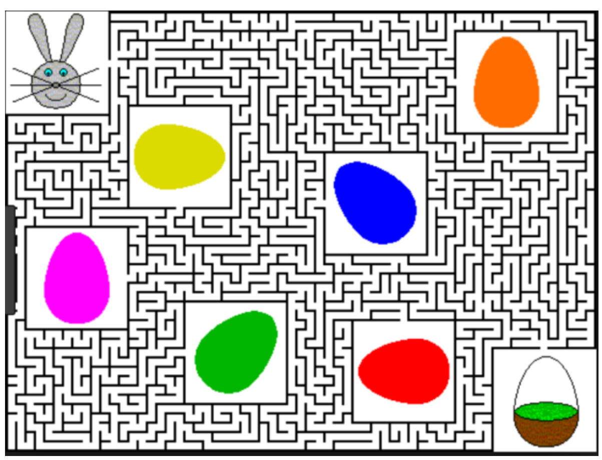
Solve: 12.1/ exercise 6 (first show f is well-defined!);

12.2/ exercises 4 (first show f is well-defined!), 6 (first show f is well-defined!), 10 (first show f is well-defined!), 12 (first show f is well-defined!), 14 (first show f is well-defined!), 16, 18 (first show f is well-defined!);

12.3/ exercise 4;

extra credit: 12.3 / exercise 6;

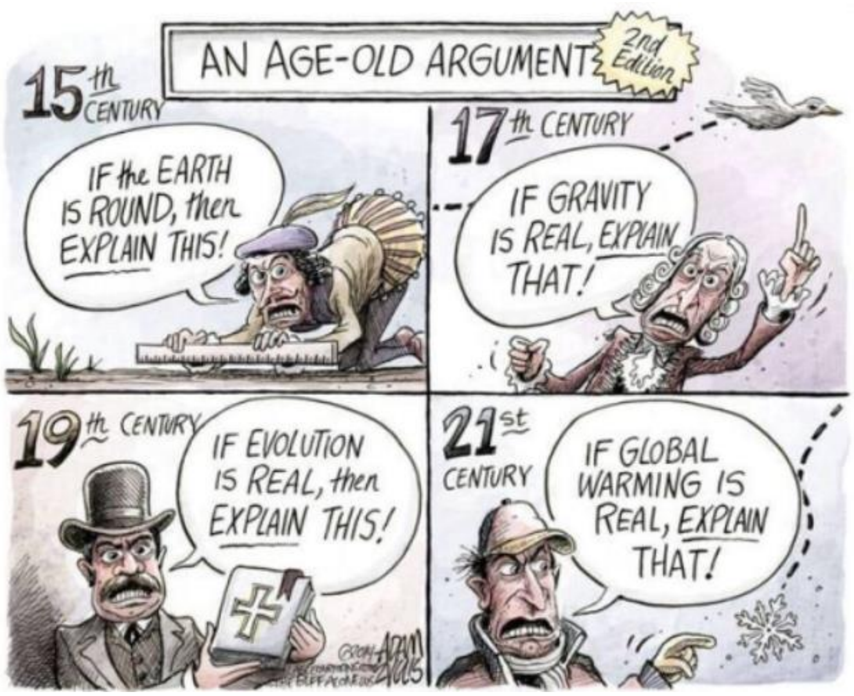
12.4/ exercise 6 (first show f and g are well-defined!) and 8 (first show f and g are well-defined!)



Homework 11: Due: Wednesday, 24 April

Review sections 12.5, 12.6. Study 10.2, 10.4, 10.5, 14.1, 14.2, 14.3

Solve: 12.5/ exercises 2, 6, 8, 10; 12.6/ exercises 2, 4, 8; 14.1/ exercises 12, 16; 14.2/ exercises 10, 12, 14; 14.3/ exercises 6, 8, 10; exercises for chapter 10 /18, 28



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