

## Errata for Game Theory: An Introduction, by E.N.Barron

Please notify me at [ebarron@luc.edu](mailto:ebarron@luc.edu) for any errors. The following list of errors is current as of February 21, 2012.

### 0.1 Errors for the 3rd Printing

I have received many notifications of errors listed below. I would like to mention especially Professor Kevin Easley and his Game Theory class, and Cecilia Dao and Prof Michael Lugo at UC Berkeley.

1. p. 9, line 5 from top, "If I passes, II gets..." should be "If I spins, II gets  $\frac{1}{36}$  and I loses  $-\frac{1}{36}$ . If I passes then II gets  $\frac{3}{2}$  and I loses  $-\frac{3}{2}$ ."
2. p. 12, line 7 from top, "II will play row 1..." should be "I will play row 1..."
3. p. 25, second line of Lemma 1.3.6 " $\forall j$ " should be " $\forall j = 1, 2, \dots, m$ ".
4. p. 31, property 2 of (1.3.1) should be  $j = 1, 2, \dots, m$ .
5. p. 38, line 2 from top ( $x^* = \frac{1}{4}, \frac{10}{4}$ ) should be ( $x^* = \frac{1}{4}, v = \frac{10}{4}$ ).
6. p. 47 Exercise 1.28 modified to  
Show that if  $(X^*, Y^*)$  and  $(X^0, Y^0)$  are both saddle points for the game with matrix  $A$ , then so is  $(X^*, Y^0)$  and  $(X^0, Y^*)$ ; so is,  $(X_\lambda, Y_\beta)$  where  $X_\lambda = \lambda X^* + (1 - \lambda)X^0, Y_\beta = \beta Y^* + (1 - \beta)Y^0$  and  $\lambda, \beta$  any numbers in  $[0, 1]$ .
7. p. 57, The second line of Equation (2.1.1) should be  $x[y(a_{11} - a_{21}) + (1 - y)(a_{12} - a_{22})] + y(a_{21} - a_{22}) + a_{22}$ .
8. p. 59, line 5,  $X$  should be  $X^*$ .
9. p. 60, line 11 from top, "...property 4..." should be "property 3..."
10. p. 66, problem 2.10(c), the answer should be  $Y^* = (\frac{1}{3}, \frac{2}{3}, 0)$ .
11. p. 72, line 13, Aaron Burr was Thomas Jefferson's vice president, not John Adams'.
12. p. 90, problem 2.32, answer on p. 390 incorrectly explains why  $Y^* = (1, 0, 0, \dots)$ . After noting that only the first and last column are used to determine and  $X^*$  with a finite number of columns, the matrix for II becomes  $\begin{bmatrix} a & 2a \\ a & \frac{1}{n} \end{bmatrix}$  and if  $n$  is large enough that  $2a > \frac{1}{n}$ , there is a pure saddle at first row, first column.
13. p. 124, line 8 from top, " $x = \frac{r}{R} =$ " should be switched with " $y = \frac{m}{M} =$ ".
14. p. 131, line 7 from bottom missing comma and right parenthesis,  $E_1(x, y) = (x, 2(1 - x)) \cdot (y, 1 - y)^T$
15. p.154, problem 3.23, the labels *straight* and *avoid* should be reversed.
16. p.160, problem 3.27(a), the labels *straight* and *avoid* should be reversed. The answer for (a) on p. 393, for the mixed Nash, should be  $X_3 = (\frac{1}{3}, \frac{2}{3}), Y_3 = (\frac{1}{3}, \frac{2}{3})$ . Also, for (b) the payoff subscripts should be I, and II, instead of 1 and 2.
17. p. 187, line 10 from bottom,  $1 - p_{II}(y)$  should be  $1 - 2p_{II}(y)$  in the case  $x > y$ .
18. p. 193, line 4 should be  $\frac{1}{4x^3}$ , not  $\frac{1}{4}x^3$ .

19. p. 227 Change Definition 5.1.2 to

Let  $x_i$  be a real number for each  $i = 1, 2, \dots, n$ , with  $\sum_i x_i \leq v(N)$ . A vector  $\vec{x} = (x_1, \dots, x_n)$  is an **imputation** if ...

20. p. 227 Remark 3. Change to:

Group rationality means any increase of reward to a player must be matched by a decrease in reward for one or more other players. Why is group rationality reasonable? Well, we know that  $v(N) \geq \sum_i x_i \geq \sum_i v(i)$ , just by definition. If in fact  $\sum_i x_i < v(N)$ , then each player could actually receive a bigger share than simply  $x_i$ ; in fact, one possibility is an additional amount  $(v(N) - \sum_i x_i)/n$ . This says that the allocation  $x_i$  would be rejected by each player, so it must be true that  $\sum_i x_i = v(N)$  for any reasonable allocation.

21. p. 228, Line 7. should read “any two n-person cooperative games.”

22. p. 228, Lemma 5.1.3, line 2. “. . . normalization with characteristic function  $v$ ” should be “. . . normalization with characteristic function  $v'$ ” with a prime on the  $v$ .

23. p. 230, Definition 5.1.5, the excess function is defined for imputations  $\vec{x} \in X$  and not just  $\vec{x} \in R$ .

24. p. 240 Exercise 5.3, should be “Given the characteristic function  $v(i) = 0, i = 1, 2, 3, 4$  and ...”.

25. p. 241, Exercise 5.9, should be “. . . and if  $x \in C(0)$ , then ...”.

26. p. 244, Last line preceding Lemma 5.1.10: text should read “. . . or there is more than one allocation in  $C(0)$ .”

27. p. 246, Line 22, The defining conditions for  $C(\varepsilon)$  should also have  $S \subsetneq N$  and  $S \neq \emptyset$ .

28. p. 249, Three lines from bottom, Should also include the condition  $S \neq \emptyset$ .

29. p. 249, line 5, “. . .  $v(S) = -(4 - |S|), v(1234) = -4, v(\emptyset) = 0, \dots$ ”

30. p. 279, line 2, Should be “. . . for some ...”

31. p. 280 Just before Definition 5.4.1. “The entire triangle in Figure 5.9 ...” should be “The entire 4-sided region ...”

32. p. 291, Example 5.26, change “utility” to “payoff.”

33. p. 291, Example 5.26. Line 7, The comma after  $\ln(y + 1)$  is misplaced to the right.

34. p. 296, Just below line 2 from top should have the sentence: “It is proved in [7] that this line *must* pass through the optimal threat security point.”

35. p. 296, Line 4 from bottom,  $m_p u^t + v^t$  should be  $-m_p u^t - v^t = \frac{3}{8} u^t - v^t$ . Line 2 from bottom also  $m_p u^t + v^t$  should be  $-m_p u^t - v^t$ .

36. p. 311, line 12 from bottom,  $\geq$  sign should be  $+$  sign.

37. p. 330, line 8, “If either  $f_{p_1} + g_{p_2} > 0 \dots$ ”

38. p. 380, Footnote, The name “Rockafeller” is misspelled and should be “Rockefeller”

39. p. 389, solution to problem 2.29, should be  $(0.9)(15) + (0.9)(30) + (0.4)(20) + (0.1)(35) = 52\%$ ., also remove one parenthesis in -line 3.

40. p. 391, solution to problem 3.2 should have  $v(B^T) = 301/65$ ,  $X^* = (\frac{5}{13}, \frac{8}{13}, 0)$ ,  $Y^* = (\frac{7}{13}, \frac{6}{13})$  and  $X^* = (\frac{5}{13}, \frac{8}{13}, 0)$  is the maxmin strategy for player II.

41. p. 395, problem 4.1, Should be  $u_i(q_1, \dots, q_i, \dots, q_N) = q_i(\Gamma - \sum_{j=1}^N q_j - c_i)$ , and

$$q_i = \frac{1}{N+1} \left( \Gamma - Nc_i + \sum_{j=1, j \neq i}^N c_j \right). \text{ If } c_i = c, i = 1, 2, \dots, N, \text{ then } q_i = (\Gamma - c)/(N+1) \rightarrow 0 \text{ as } N \rightarrow \infty.$$

## 0.2 Errors for the 1st and 2nd printing

1. Stephen Conwill found the following errors.

(a) p.8 In the table at the bottom of the page II3 should be the strategy: *If I1, then S; If I2, then S*. The strategy II4 should be: *If I1, then S; If I2, then P*.

(b) p. 9 line 5 from the top “pass as well” should be *spin*.

2. Dinesh Ayyappan found the following error.

(a) p.11 line 5 from top, the word “largest” should be replaced by the word “smallest”.

3. p. 12 Lemma 1.1.3, second line of proof should be

$$v^+ = \min_j \max_i a_{i,j} \leq \max_i a_{i,j^*} \leq a_{i^*,j^*} \leq \min_j a_{i^*,j} \leq \max_i \min_j a_{i,j} = v^-.$$

p. 12 in proof of Lemma 1.1.3, “Let  $i^*$  be such that . . .  $j = 1, 2, \dots, m$ . Should be: “Let  $j^*$  be such that  $v^+ = \max_i a_{i,j^*}$  and  $i^*$  such that  $v^- = \min_j a_{i^*,j}$ . Then

$$a_{i^*,j} \geq v^- = v^+ \geq a_{i,j^*}, \text{ for any } i = 1, 2, \dots, n, j = 1, 2, \dots, m.$$

4. p. 16, line 6,  $v^+ = \min_{x \in C} \max_{y \in D} f(x, y)$ , and  $v^- = \max_{y \in D} \min_{x \in C} f(x, y)$ , should be

$$v^+ = \min_{y \in D} \max_{x \in C} f(x, y), \text{ and } v^- = \max_{x \in C} \min_{y \in D} f(x, y).$$

5. p. 22, The last line of the third paragraph “These probability vectors are called mixed strategies, and will turn out to be the class correct class of strategies for each of the players.” should be “These probability vectors are called mixed strategies, and will turn out to be the correct class of strategies for each of the players.”

6. p. 31, last line, remove “).”

7. p. 34, line 3 from top, “...property 3,...” should be “...properties 3 and 5...”

8. The following errors were found by Yan Jin .

(a) p. 43, line 12 from bottom,  $E(4, Y) = -5y + 6(1 - y)$  should be  $E(4, Y) = 7y - 8(1 - y)$ .

(b) p. 44, line 1 from top,  $E(1, X)$  should be corrected as  $E(X, 1)$ . Line 2 from top,  $E(4, X)$  should be  $E(X, 2)$ , and  $(x = 5/6, 1/3)$  should be corrected as  $(x = \frac{5}{6}, v = \frac{1}{3})$ .

9. p. 47, Problem 1.29, part (a) should have  $\min_j E(X, j) = -\frac{42}{9}$ .

10. p. 55, Quotation added

11. The following errors were also found by Yan Jin

(a) p. 68, the second line of the proof of Theorem 2.3.1 should read

$$E(X, X) = XAX^T = -XA^T X^T = -(XA^T X^T)^T = -XAX^T = -E(X, X).$$

In other words, the third  $A$  should be  $A^T$ .

(b) p. 69, the third line from the bottom,  $(a\lambda, -b\lambda, c\lambda)$  should be  $(c\lambda, -b\lambda, a\lambda)$ .

12. p. 90 problem 2.34, Remove part (a) and (b) and change hint to:

**Hint:** Player I has 4 strategies, e.g., If ace, bet 2; If jack, bet 2. Player II also has 4 strategies, e.g., If I bets 4, then Fold; If I bets 2, then Call. Player I's strategies are then,  $(2, 2), (2, 4), (4, 2), (4, 4)$ , where the first number is the amount to bet if an ace. Player II's strategies are  $(F, C), (C, C), (F, F), (C, F)$ , where the first letter is for a bet of 4.

13. p. 111, line 7 from top  $E_2$  should be  $E_{II}$ .

14. Joe Condon found the following errors.

(a) p.117 line 1,  $v(B^T) = \frac{1}{4}$  should be  $v(B^T) = \frac{3}{4}$ . In line 8 from the top,  $X^{B^T} = (\frac{1}{4}, \frac{3}{4})$  should be  $X^{B^T} = (\frac{3}{4}, \frac{1}{4})$ . Line 9 from the top "value of  $\frac{1}{4}$ " should be "value of  $\frac{3}{4}$ ."

(b) p.123, The case  $R < 0$  should have the possible solutions

$$\text{if } y = 0 \implies 1 \geq x \geq \frac{r}{R},$$

$$\text{if } 0 < y < 1 \implies x = \frac{r}{R},$$

$$\text{if } y = 1 \implies 0 \leq x \leq \frac{r}{R}.$$

In addition, in line 2 from the bottom  $R < 0$  should be  $R > 0$ , the figure 3.2 should have  $R > 0$ , and line 2 above the figure should have  $R > 0$ .

15. p. 125, line 9 from bottom,  $E(1, Y)$  should be  $E_I(1, Y)$ .

16. p. 137, Matrix B, first row should be  $[0, 1, -1]$ , not  $[0, 2, 1]$ . Then  $value(B^T) = \frac{2}{3}, X_B = (\frac{1}{3}, \frac{2}{3}, 0), Y_B = (\frac{2}{3}, \frac{1}{3})$ . Fu-Te Hsu kindly pointed this out.

17. p. 145, line 5 from bottom,  $Y^*T$  should be  $Y^{*T}$ .

18. p. 154, problem 3.23 has the answer fixed on p. 393: should have  $f(x, y, p, q) = 7x + 7y - 6xy - 6 - p - q$ , and  $2 - x \leq q$  should be  $2x - 1 \leq q$ . Problem 3.25 should read "Find as many as you can by...."

19. p. 176, line 3 from top, replace  $Q$  by  $q$ .

20. p. 181, line 3 from bottom, should have  $u_1(q_1^*, q_2^*) = \frac{(\Gamma - 2c_1 + c_2)^2}{8}$ .

21. p. 182, line 2 from bottom, replace  $\gamma$  by  $\Gamma$ . Line 4 from the top should be  $u_1(q_1^*, q_2^*) = \frac{(\Gamma - 2c_1 + c_2)^2}{8}$ .

Line 8 from the bottom should be  $u_1(q_1^*, q_2^*) = \frac{(\Gamma - c)^2}{8}$ .

22. p. 184, line 10 from bottom,  $q_1$  should be  $q_1^0$  in three spots.

23. p. 185, (i) Problem 4.3 should be changed to "Compare profits for firm 1 in the model with uncertain costs and the standard Cournot model. Assume  $\Gamma = 15; c_1 = 4; c^+ = 5; c^- = 1$  and  $p = 0.5$ ."

(ii) Problem 4.6 : Should be: Suppose that two firms have constant unit costs  $c_1 = 2, c_2 = 1$ , and  $\Gamma = 19$  in the Stackelberg model.

24. p. 194, Problem 4.17, line 7 from bottom, "... each variable separately ..." should be "... the variable they control ..."
25. p. 221, Example 5.1(4): "but will take \$1 million ...," should be "but will take \$100 million ..."
26. p. 224, In matrix for player 3 versus coalition players 12, entry for  $A$  versus  $AB$  should be +2.
27. p. 225, line 9 from bottom, "...and the assistance of player 1 doesn't help since  $v(13) = -1$ ." should be "but the assistance of player 1 does help since  $v(13) = 1$ ."
28. p. 227, Professor Kevin Easley kindly pointed out the following error: The first sentence of Definition 5.1.2 should be replaced by "Let  $x_i$  be a real number for each  $i = 1, 2, \dots, n$ "
29. p. 227, line 12 from bottom,  $\sum_i v(i) \geq \sum_i x_i$  should be  $\sum_i x_i \geq \sum_i v(i)$
30. p. 240, Problem 5.2(b) solution  $\frac{38}{5}$  should be  $\frac{22}{5}$ .
31. p. 241, Problem 5.10:  $x - 2$  should be  $x_2$ .
32. p. 243, first line of second paragraph, "...give away to the whomever ..." should be "...give away to whomever ..."
33. p. 244, line 2 from top,  $x$  should be  $\vec{x}$  in two places.
34. p. 245, Remark 3,  $e(S, \vec{x})$  missing in two spots. Last line should have  $\vec{x}$  and  $\vec{x}^*$ .
35. p. 246,  $x_1 + x_2 + x_3 = \frac{5}{2}$  should be  $x_1 + x_2 + x_3 = 5/2$ .
36. p. 253, line 15 from top,  $\frac{9}{10}$  should be  $9/10$ .
37. p. 259, line 6 from bottom,  $\frac{11}{12}$  should be  $11/12$  in three places.
38. p. 260, line 3 from bottom,  $c_{13}$  should be 18, not 10.
39. p. 265, Last paragraph before Example 5.14 should have a last sentence: At the end of this chapter you can find the Maple code to find the Shapley value.
40. p. 275, line 4 from top,  $n$  should be 4.
41. p. 306, The Maple code for the calculation of the Shapley value is added.
42. p. 388, Problem 2.21 should have solution  $X^* = (0, \frac{5}{11}, \frac{5}{11}, 0, \frac{1}{11}) = Y^*$ .
43. p. 393, Problem 3.24,  $Y_1 = (\frac{5}{13}, \frac{5}{13}, \frac{2}{13})$  should be  $Y_1 = (\frac{6}{13}, \frac{5}{13}, \frac{2}{13})$ .
44. p. 394, Problem 3.27(c),  $Y_1 = (\frac{5}{13}, \frac{5}{13}, \frac{2}{13})$  should be  $Y_1 = (\frac{6}{13}, \frac{5}{13}, \frac{2}{13})$ .
45. p. 395, Problem 4.3 should have the answer "Profit for firm 1 is 10, compared with 16 or 7.11 if  $c_2 = 5$  or  $c_2 = 1$ , resp.."
46. p. 400 Problem 5.9 "since  $-x - 1$  ..." should be "...since  $-x_1$  ..." . Problem 5.13 should have  $16 - x_1 - x_2$ , not  $16 - x_1 - x - 2$ .
47. p. 401, Problem 5.19 solution in (b) should have  $x_4 = \frac{3}{2}$ , not 32.
48. p. 402, Problem 5.20, line 1 "The characteristic function for the savings game is ..."
49. p. 404-405, Problem 6.5 should have solutions (b) and (c) switched.
50. p. 409, Reference [7], should be *Game Theory: Mathematical Models of Conflict*