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Primary fields and boundedness of smeared primary fields.

Start with two irreducible positive energy representations H_λ and H_μ of LG of level l. Look at the $\mathrm{Hom}_{L_IG}(H_\lambda, H_\mu)$, where $I \subset S^1$. Remark: Hom consists Goal: Construct explicit elements of this hom. Remark: Abstract theory tells us that the hom is nonzero.

Example: $G = \mathrm{SU}(n)$, $V = \mathbf{C}^n$ is the vector representation, $H = L^2(S^1, V)$. There is an action of $\mathrm{Cliff}(H)$ on F_V. There is a sequence $LG \rtimes \mathbf{T}_r \to \mathrm{U}_r(H) \to \mathrm{PU}(F_V)$. We have $H_\lambda, H_\mu \subset F_V^{\otimes l}$. If $f \in L^2(S^1, V)$, then $a(f) \in B(F_V^{\otimes l})$ is a creation operator.

We have $\pi(g)a(f)\pi(g)^* = a(gf)$ for any $g \in \widetilde{LG} \rtimes \mathbf{T}_r$. Define $\phi(f) = P_\mu a(f) P_\lambda^* \in \mathrm{Hom}^b(H_\lambda, H_\mu)$. The same relation is true for $\phi(f)$ instead of $a(f)$. If $g \in L_IG$ and $\mathrm{supp}(f) \subset I^c$, then $gf = f$. Hence $\phi(f) \in \mathrm{Hom}_{L_IG}(H_\lambda, H_\mu)$. Moreover, $\|\phi(f)\| \leq \|f\|$.

Fields are maps from functions on S^1 with values in V to $\mathrm{Hom}(H_\lambda, H_\mu)$. Primary means that the map is $LG \rtimes \mathbf{T}_r$-equivariant.

Definition: V is a G-module, H_λ and H_μ are irreps of level l of G. Define $V^f = V[z, z^{-1}]$, which is acted upon by $L^p g_{\mathbf{C}} \rtimes i\mathbf{R}d$.

A primary field of charge V and level l is a linear map $\phi\colon V[z, z^{-1}] \otimes H_\lambda^f \to H_\mu^f$, which is $L^p g_{\mathbf{C}} \rtimes i\mathbf{R}d$ equivariant.

Modes of the primary field ϕ are $\phi(v, n) = \phi(v \otimes z^n)\colon H_\lambda^f \to H_\mu^f$. Properties (equivalent to equivariance): (1) $[X(n), \phi(v, n)] = \phi(Xv, m+n)$; (2) $[d, \phi(v, n)] = -n\phi(v, n)$. Here $X \in g_{\mathbf{C}}$ and $X(n) = X \otimes z^n \in L^p g_{\mathbf{C}}$. (2) imples that $\phi(v, n)$ lowers energy by n: $\phi(v, n)\colon H_\lambda(k) \to H_\mu(k-n)$. In particular, $\phi(v, 0)\colon H_\lambda(0) \to H_\mu(0)$. $\phi_0\colon V \otimes V_\lambda \to V_\mu$ is the initial term of ϕ. We have $\phi_0 \in \mathrm{Hom}_G(V \otimes V_\lambda, V_\mu)$.

Lemma: The map $\phi \mapsto \phi_0$ is injective.

Proof: H_λ^f is generated by $H_\lambda(0)$ as a $L^p g_{\mathbf{C}}$-module.

Proposition: This map is an isomorphism if λ, μ, and ν are admissible at level l and at least one of them is minimal (highest weight is dominant) G-module. (For $\mathrm{SU}(n)$ minimal means that the representation is the exterior power of the standard representation.)

Vector primary fields of $G = \mathrm{SU}(n)$. If f and g are signatures of admissible G-module of level l, then we want to study $\mathrm{Hom}_{\mathrm{SU}(n)}(V \otimes V_f, V_g)$. $V \otimes V_f = \oplus_{g=f+1} V_g$. Here $g = f+1$ means that g can be obtained from f by adding one box.

Consider $W = V \otimes \mathbf{C}^l$. We have $\Lambda W = (\Lambda V)^{\otimes l}$. $S\colon W \otimes \Lambda W \to \Lambda W$ is the exterior multplication. We have $V_f, V_g \subset (\Lambda V)^{\otimes l}$.

Lemma: Let $T \in \mathrm{Hom}_{\mathrm{SU}(n)}(V \otimes V_f, V_g)$, $T \neq 0$. We can find $\mathrm{SU}(n)$-equivariant projections $P\colon W \to V$, $P_f\colon \Lambda W \to V_f$, $\P_g\colon \Lambda W \to V_g$ such that $T = P_g S(P^* \otimes P_f^*)$.

Proof: ...

Theorem: Any $\mathrm{SU}(n)$-intertwiner $\phi_0\colon V \otimes H_\lambda(0) \to H_\mu(0)$ is the initial term of a unique vector primary field. All vector primary fields arise as compressions of fermions and satsify $\|\phi(f)\| \leq c\|f\|_2$ for all $f \in V[z, z^{-1}]$. The map extends continuously to $L^2(S^1, V)$ and satisfies the global equivariance relation $\pi_\mu(g)\phi(f)\pi_\lambda(g)^* = \phi(gf)$.

Construction: $\Lambda W = F_W(0)$, $W \subset L^2(S^1, W)$ (constant functions). $V_\lambda, V_\mu \subset W$, $H_\lambda, H_\mu \subset F_W$ give us projections P_λ, P_μ. $\phi_{\lambda,\mu}(v, n) = P_\mu a(v \otimes z^n) P_\lambda^*$.