Orit Davidovich.

Modularity of the category of representations of conformal nets.

Definition: A conformal net is a cosheaf of von Neumann algebras on a circle that satisfies properties expected of algebra of local observables which is covariant with respect to $\mathrm{PSU}(1,1)$.

Example: Vector conformal net associated to a loop group.

Definition: A representation of conformal net is a compatible family of representations of corresponding algebras.

Example: Every PER gives rise to a representation of vector conformal net associated to the loop group.

Extra assumptions: (1) Separability; (2) Split property; (3) Strong additivity.

For loop nets: (1) Obvious; (2) and (3) proved in Wassermann's paper.

Split property. Equivalent formulations: (1) If $\bar{I} \cap \bar{J} = \emptyset$, then $A(I) \otimes A(J) \to A(I) \vee A(J) \subset B(H)$ is an isomorphism; (2) If $I \subset\subset J$, then $A(I) \to A(J)$ factors through a type I factor.

We are looking at separable unital representations (Hilbert spaces are separable and identity acts by identity).

We consider the category of representations of conformal net A with finite μ_2-index. It turns out that this category is a modular tensor category.

Goal for today's talk: The category of (separable) representations of a conformal net is semisimple (there are finitely many irreducible separable representations and every representation is completely reducible).

Localization: A representation π of A is localized at $I_0 \subset S^1$ if the following two conditions are satisfied: (1) π is defined on H_0 (H_0 is always vacuum, the defining representation of a conformal net). (2) The restriction of π to $A(I_0')$ is trivial.

Proposition: Let π be a representation of A. Then for any interval $I_0 \subset S^1$, π is equivalent to a representation of A localized in I_0.

Proof: All values of A on intervals are type III_1 factors, which are simple as algebras and every representation is continuous in the σ-strong operator topology.

Any two representations of type III factor are unitarily equivalent. Thus we can force $\pi_{I_0'}$ to be the trivial representation.

Note: If ϕ is local at I_0, then $\phi(A(I)) \subset A(I)$ for every $I \supset I_0$.

Proof: Haag duality: $A(I) = A(I')'$. $\phi_I(A(I))$ commutes with $A(J') = \phi_{I'}(A(J'))$. Use locality in A and regularity.

Remark: Every representation of A localized at I gives a representation of $A(I)$ but not vice versa.

Dimension: If π is an irrep of A, we have an inclusion of type III factors $\pi(A(I)) \subset \pi(A(I'))'$.

Proposition: The index $[\pi(A(I'))' : \pi(A(I))] \in [1, \infty]$ does not depend on I.

Heuristic justification: Index should only depend on equivalence class and since π is equivalent to a representation localized in any I, the index should not depend on I.

Definition: The statistical dimension of π is defined to be the square root of the above index.

Theorem (Longo): The statistical and the quantum dimensions are the same.

2-interval inclusions: Suppose $\bar{I} \cap \bar{J} = \emptyset$, $E = I \cup J$, $A(E) = A(I) \vee A(J) \subset B(H_0)$. We have an inclusion of type III factors $A(E) \subset A(E')'$.

Proposition: $[A(E')' : A(E)]$ does not depend on E.

Definition: $\mu_2(A)$ is the above index. (It is equal to the square of the global dimension.)

Claim: Given extra assumptions on the conformal net such that $\mu_2(A)$ is finite implies that $\mu_2(A)$ gives an upper bound on the number of equivalence classes of irreps in the category of representations.

Complete reducibility: Take the colimit of a conformal net and obtain a C*-algebra $C^*(A)$.

Disintegration theory: If π is a separable non-degenerate representation of $C^*(A)$ on H, then H can be decomposed as the direct integral of some representations π_x on H_x over some measurable space X.

Sketch of a proof: (1) Central decomposition; (2) Show all π_x have type I; (3) π_x are multiples of an irreducible; (4) If $x \neq x'$, then π_x and $\pi_{x'}$ are not multiples of the same irreducible; (5) Only finitely many irreps ($\mu_2 < \infty$), hence $|X$ is finite and direct integral is actually a direct sum.