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Braiding of smeared primary fields.

We hawe two groups: $B_n = \pi_1(\mathrm{Conf}_n(\mathbf{C}))$ and P_n (loops in the configuration space of n distinguished points in the plane).

Our picture: $\mathbf{CP}^1 = S^2$.

In order to get configurations of points, we fix ∞, and a tangent vector. The complement is $\mathbf{C}$ with a preferred framing.

4 point functions take as input 4 points on $\mathbf{CP}^1$ together with IPERs assigned to those points, elements of those IPERs, and tangent vectors at the points, and producte elements of a vector space.

Geometric picture of fusion: If two points with IPERs collide, then the corresponding IPERs fuse.

Why 4 points? There is an action of $\mathrm{PGL}_2(\mathbf{C})$ on $\mathbf{CP}^1$ that is sharply triply transitive.

A function on configuration of 4 points in the space is a function on $\mathbf{CP}^1 \setminus \{0, 1, \infty\}$.

In fact, we will have a multivalued function: 4 point functions will satisfy the KZ equation and solutions don't necessarily exist globally.

Anatoly mentioned a correspondence between differential equations of regular singular type, local systems (locally constant sheaves), and representations of fundamental group of configuration space.

Pushing z around endows the space of 4 point functions at some fixed configuration with an action of $\pi_1(\mathbf{CP}^1 \setminus \{0, 1, \infty\}) = F_2$.

Primary field picture: We are given some representations U and V of $\mathrm{SU}(n)$, positve energy representations H_i, H_j, H_k of $\mathrm{LSU}(n)_l$. Primary field is an equivariant map $\phi\colon V[z, z^{-1}] \otimes H_j \to H_i$. Primary fields can be smeared by integration against a smooth function, i.e., if f is a smooth function $f = \sum f_n z^n$, then $\phi^\vee_{j,i}(v, f) = \sum_n \phi^\vee_{j,i}(v, n) f_n$

To describe the braiding, we compare $\phi^u_{k,j}(u, g)\phi^\vee_{j,i}(v, f)$ with $\phi^\vee_{k,h}(v, f)\phi^u_{h,i}(u, g)$.

The formula: $\phi^U_{k,j}(u, f)\phi^V_{j,i}(v, g) = \sum_h c_{j,k}\phi^V_{k,h}(v, e_{\mu_{j,h}} g)\phi^U_{h,i}(u, e_{-\mu_{j,h}} f)$.

What is $\mu_{j,h}$? It is a real number, determined by some subset of U, V, H_i, H_j, H_k, H_h.

Sugawara construction produced an action of Virasoro algebra on H_i etc. There is a distinguished operator L_0 (energy). PERs are graded by L_0.

Each IPER has L_0 eigenvalues in some coset of $\mathbf{Z}$, and there is a well-defined lowest energy. If L_0 acts by integers, then the action of L_0 integrates to an action of rotation circle.