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The symmetric monoidal 3-category of conformal nets.

Motivation: TQFTs are functors from 1-category of $(n-1)$-manifolds and n-bordisms to Hilbert spaces and bounded maps.

Theorem (Bartels-Douglas-Henriques): There is a SM 3-category CN whose objects are conformal nets.

Theorem (BD, HL): framed local C-valued n-dimensional TQFTs are in bijection with dualizable objects in C.

Theorem (BDH): A conformal net A is dualizable if it is a direct sum of irreducible conformal nets with finite μ-index.

Warm-up: Algebras and bimodules.

Upshot: Algebras and bimodules form a category object in the 2-category of symmetric monoidal categories.

The 3-category of conformal nets is a bicategory object in the 2-category of symmetric monoidal categories.

Conformal nets (revisited).

Definition: Int the topological category whose objects are oriented intervals and morphisms are embeddings (not necessarily orientation preserving) (with topology of pointwise convergence).

Definition: A conformal net is a continuous functor A from Int to the category of von Neumann algebras satisfying

(a) If $\phi: I \to I$ is a diffeormophism that restricts to identity in a neighborhood of the boundary of I, then $A(\phi)$ is inner.

(b) If ϕ is orientation preserving/reversing, then $A(\phi)$ is a homomorphism/antihomomorphism.

Conformal nets and 2-algebras. $A = A([0,1])$. i and j are two inclusions of $[0,1]$ into $[0,2]$. They equip A with a new "vertical" multiplication: $\mu(x,y) = s_*(i_*(x), i_*(y))$.

Claim: There exists $v \in A$ such that $v\mu(\mu(x,y),z)v^{-1} = \mu(x,\mu(y,z))$ and $v^2 = \mu(1,v)v\mu(v,1)$.

1-morphisms are defects, 2-morphisms are sectors, 3-morphisms are morphisms of sectors.

Definition: A bicolored interval is an interval I with 2 subintervals I_w and I_b with $I = I_w \cup I_b$ such that $I = I_w \cup I_b$ (or one of the intervals is empty) together with a function from the neighborhood of $I_w \cap I_b$ to $\mathbf{R}$. We can define a category consisting of these intervals.

Definition: A defect $D: A \to B$ is a cosheaf $D: \mathrm{Int}_{\mathrm{bc}} \to \mathrm{vNa}$ such that restrictions of D to the subcategories of black and white intervals are A and B respectively.

Sectors: Consider intervals I in S^1 such that either $i \notin I$ or $-i \notin I$.