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Boundary CFTs and classification via Frobenius algebras.

Locality: In 1D: If $I \cap J = \emptyset$, then $[A(I), A(J)] = 0$. In 2D: Replace $I \cap J = \emptyset$ by a different condition: One region lies in the causal complement of the other region.

Assume complete rationality: (1) Split property: $I \cap J = \emptyset$ implies $A(I) \vee A(J) = A(I) \otimes A(J)$. (2) Strong additivity: $A((a,b)) \vee A((b,c)) = A((a,c))$. (3) Finite index: μ_2 is finite.

Denote by M_+ the positive Minkowski space $\{(t,x) \mid x > 0\}$.

A boundary CFT over a one-dimensional conformal net A is an assignment $O \mapsto B_+(O) \subset B(H)$ that is local, isotonic, and satisfies the following properties: (1) There is a unique vacuum vector Ω in $B(H)$. (2) Covariance: $U(g)B_+(O)U(g)^* = B_+(gO)$ when gO is a double cone. (3) An action π of A on H. Covariance: $U(g)\pi(A(I))U(g^*) = A(g(I))$. (4) Joint irreducibility: Denote by $\pi(A)''$ the von Neumann algebra generated by all $\pi(A(I))$. We have $\pi(A)'' \vee B(O) = B(H)$.

Examples: Trivial BCFT over A: $O \mapsto A_+(O) := A(I) \vee A(J)$. Dual to the trivial: $O \mapsto A_+^*(O) := A(K)' \cap A(L)$.

Fix A. A BCFT over A gives a Haag-dual BCFT over A, which gives chiral extensions of A (1d conformal nets, extend A), which gives "superfactors" of A (of index μ_2), which gives Frobenius algebras in some category coming from A.

Definition: Given BCFT B_+ over A its boundary net $(B)^g$ is given by $B^g(I) = B_+(W_I)$. Possibly non-local, relatively local with respect to A: $[A(I), B(J)] = 0$.

Definition/Theorem: Given irreducible non-local chiral extension B of A, the induced BCFT is $B_+^i(O) := B(L) \cap B(K)'$.

Check: $(B_+^i)^g = B$, $(B^g)_+^i = B^d$.

Given chiral extensions $I \to B(I) \supset A(I)$. Fact: (1) We have a consistent family of conditional expectations $E_I \colon B(I) \to A(I)$. (2) If irreducible and finite index, then E_I implemented by $E \colon B \to A$.

Reeh-Schlieder property: Ω is cyclic and separating for any $B(I)$. Theorem: Classifying chiral extensions B of A. is the same as classifying "extensions" of $A(I)$.

We now describe "some category coming from A".

Objects are $\mathrm{End}(A)$. Morphisms are intertwiners: $a \in A$ such that $a \in (\rho, \sigma)$ and $a\rho(x) = \sigma(x)a$.

A Frobenius algebra in C is an object Q, a monoid and a comonoid over this object, together with the relation $(m \otimes 1)(1 \otimes \Delta) = \Delta m$.

Example: G is a finite group. $\mathbf{C}[G]$ is a Frobenius algebra.

Example: Subfactors.

Canonical endomorphisms: $N \subset M$ is a morphism of type III factors. We define a morphism $\gamma \colon M \to N$ in the following way: $\gamma(x) = J_N J_M x J_M^* J_N \in N$, where J denotes respective modular conjugations.

Given a Frobenius algbra in $\mathrm{End}(M)$ it gives a subfactor. $E \colon M \to N$ by $E(x) := m\rho(x)\Delta$. Proof: This is a bimodule map, whose image is an algebra. $E(xE(y)) = E(x)E(y)$.