André Henriques.

Suppose A, B, C, U, and V are irreducible representations of $\mathrm{SU}(n)$. U and V are minuscule (exterior powers of vector representation in case of $\mathrm{SU}(n)$). $\mathrm{Hom}(- \otimes V, -)$ is at most one-dimensional.

We want to understand $H \otimes H_f$. Also need to understand this product for tensor powers of H instead of H.

We consider the fusion tensor product $H \otimes H_f$.

Elements of the form $\sum_n \pi(h^n)a^n$, where $a^n \in \mathrm{Hom}_{\widetilde{L_{I'}G}}(H_0, H)$ and $\pi(h^n) \in \mathrm{Hom}_{\widetilde{L_{I'}G}}(H, H)$ are dense in $\mathrm{Hom}_{I'}(H_0, H)$.

Now $\|x \otimes y\|^2 = \langle x^* x y, y\rangle = \langle \pi_f(x^* x) y, y\rangle$.

Technical theorem (transport formula): $\pi_f(a_0^* a_0) = \sum_{g=f+1} \lambda_g a_{g,f}^* a_{g,f}$.

Conclusion: The map $H \otimes H_f \to \oplus_{g=f+1} H_g$ gien by $x \otimes y \mapsto \oplus_{g=f+1} \sqrt{\lambda_g} a_{g,f} y$ is an isometry, hence it is injective. It is surjective because $\lambda_g > 0$.