The Bisognano-Wichmann theorem and nets on $\mathbf{R}^4$.

1. Minkowski spacetime.
2. Axioms for QFT.
3. BW-theorems.
4. BW and physics.

1. Spacetime. Minkwoski spacetime: $\mathbf{R}^4$ with signature $+---$.

History: 1830s (Faraday): Concept of fields (sections of bundle over space and time). Maxwell: Field equations (set of hyperbolic PDEs on the fields). Hyperbolicity: Perturbation of one field propagates with finite speed. PDEs are invariant under the Lorentz group. Einstein: Promote the above two properties to axioms. Minkowski: 4-dimensional spacetime continuum, notion of future/past/present remains objective. Axiom: There exists a preferred class of coordinates (the inertial coordinates) such that in these coordinates the components of metric form a diagonal matrix with entries ± 1.

The Poincaré group is defined as the group of diffeomorphisms preserving the Lorentz structure. Subgroups of the (time preserving) Lorentz group: Rotations of spacelike slices (SO(3)) and boosts.

2. Axioms for QFT. What is physics? Naively it is a pairing $(A, \omega) \to \omega(A)$, where A is an observable and ω is a state.

Pure states: We have perfect knowledge of the configuration. Assume that the set of observables is inside a C*-algebra. Pure states are extremal points of the set of all states.

Quantum physics: Algebras become noncommutative and the standard deviation is nonzero even for pure states because of the noncommutativity.

Observable algebras and Haag-Kastler axioms.

If O is a region in the spacetime, then O' denotes its causal complement. We consider a net of W*-algebras on the spacetime. Isotony: If $O_1 \subset O_2$, then $R(O_1) \subset R(O_2)$. Locality: If $O_1 \subset O_2'$, then $R(O_1) \subset R(O_2)'$. Poincaré symmetry: $\alpha_g(R(O)) = R(gO)$.

Vacuum representation. A representation of a Haag-Kastle net is called vacuum if: There is a strongly continuous reprsentation U of P_+ that implements α_g: $U(g)\pi_0(R(O))U(g)^* = \pi(\alpha_g(R(O)))$. Also the spectrum of the generators of $U|_{\mathbf{R}^4}$ must lie in the forward light cone. There exists a unique up to phase vacuum vector Ω that is univariant under $U|_{\mathbf{R}^4}$.