

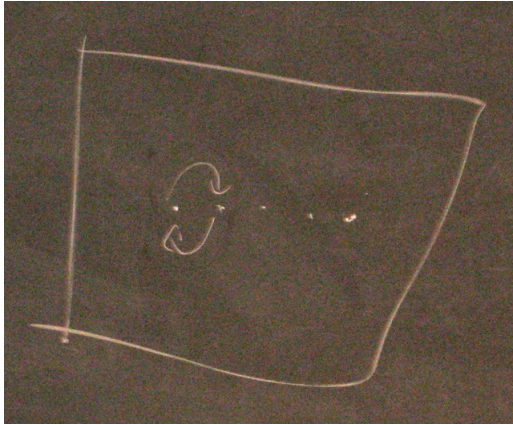
BRAIDING OF SMEARED PRIMARY FIELDS

SPEAKER: SCOTT CARNAHAN
TYPIST: MICHAEL HARTGLASS

ABSTRACT. Notes from the “Conformal Field Theory and Operator Algebras workshop,” August 2010, Oregon.

Start by discussing braiding:

When we see braiding, we have a braid group $B_n = \pi_1(\text{Conf}_n(\mathbb{C}))$. This contains P_n the set of loops in the configuration space of n distinguished points in the plane.



Date: August 20, 2010.

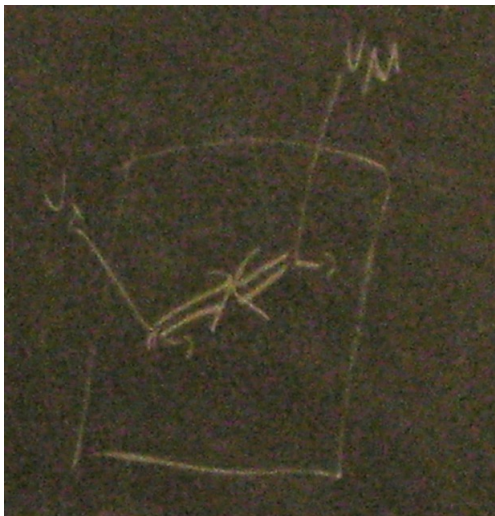
Available online at <http://math.mit.edu/~eep/CFTworkshop>. Please email eep@math.mit.edu with corrections and improvements!



In order to get configurations of points, we fix ∞ and a tangent vector. The complement is \mathbb{C} with a preferred framing.

4 point functions: Take as input 4 points on $\mathbb{C}P^1$ together with *IPER* (irreducible positive energy representations) assigned to those points. Elements of these *IPER*'s and tangent vectors at the points and produce elements of a vector space. Call this the space of 4 point functions.

Geometric picture of fusion:



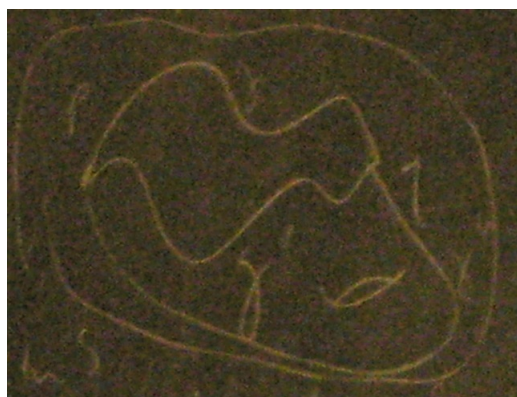
When they collide, the function that gets assigned is the function that gets attached to a single *IPER* at some point. This is the fusion object.

Why do we look at 4 points? There is an action of $PGL_2(\mathbb{C})$ on $\mathbb{C}P^1$ that is sharply triply structure i.e. given $a, b, c \in \mathbb{C}$ there is a unique $\gamma \in PGL_2(\mathbb{C})$ such that $\gamma(a) = 0$, $\gamma(b) = 1$ and $\gamma(c) = \infty$. If we have 4 points, we can send the first three to 0, 1 and ∞ and the fourth goes to a unique element called the cross ratio. In the second picture, if $\alpha = 1$, z describes the set of these points up to isomorphism.

Definition. A configuration of 4 points in the sphere is a function on $\mathbb{C}P^1 \setminus \{0, 1, \infty\}$.

In fact, we will have a multi-valued function: 4 point functions will satisfy the *KZ* equation and the solutions don't necessarily exist globally.

Toly mentioned a correspondence between differential equations of regular singular type and local systems (or locally constant sheaves). The latter corresponds to representations of the fundamental group of the configuration space by assigning to each point a vector space $p \rightarrow P_v$. To each path γ connecting p and q we get an isomorphism $V_p \rightarrow V_q$ dependent on the homotopy group.



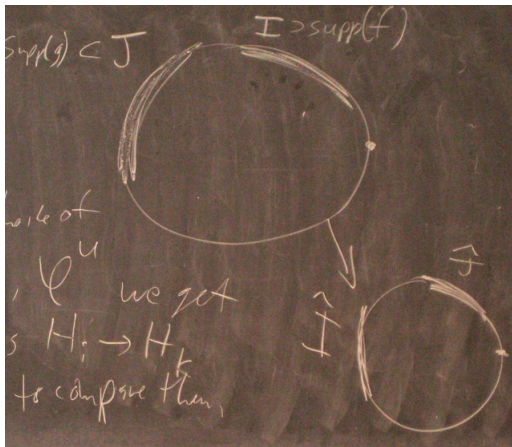
Pushing z around endows the space of 4 point functions at some fixed configuration with an action of the fundamental group of $\mathbb{C}P^1 \setminus \{0, 1, \infty\}$.

Now onto primary fields: We are given V and U representations of $SU(N)$ and H_i, H_j, H_k , positive energy reps of $LSU(N)$ primary field. $LSU(N)$ is

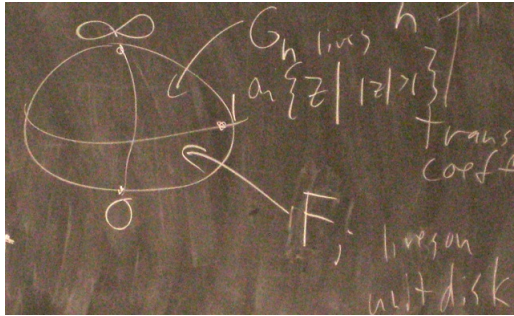
the equivalent map $V[z, z^{-1}] \otimes H_j \rightarrow H_i$. Primary fields can be “smeared” by integrating against a smooth function.

Suppose f is a smooth function $f = \sum_n f_n z^n$ then $\varphi'_{ji}(v, f) = \sum \varphi_{ji}(v, n) f_n$. This eats z^n and produces a map $H_j \rightarrow H_i$.

To describe the braiding, we compare $\varphi_{kj}^u(u, g)\varphi_{ji}^v(v, f)$ with $\varphi_{kh}^u(v, f)\varphi_{ki}^u(u, g)$.



The formula: $\varphi_{kj}^u(u, f)\varphi_{ji}^v(v, g) = \sum_n c_{jk} \varphi_{kh}^v(v, e_{\mu_{jh}} \cdot g)\varphi_{hi}^u(u, e_{-\mu_{jh}} \cdot f)$ where the c_{ij} are the transport coefficients.



$e_{\mu_{jh}}$ is a multiplicity phase $e_{\mu}(f e^{i\theta}) = e^{i\mu\theta} f(e^{i\theta})$. What is μ_{jh} ? It is determined by the representations V, U , and a subset of H_i, H_k, H_i and H_h . From Nick's talk, the Sugawara construction produced a construction of the Virasoro algebra (which is the span of $\{L_n\}_{n \in \mathbb{Z}} \oplus \mathbb{C}k$) on H_i, H_j etc. There is a distinguished operator L_0 . PER's are graded by L_0 eigenvalue $\in \mathbb{C}$. Each *IPER* has L_0 eigenvalues in some coset of \mathbb{Z} and there is a well defined lowest energy. If L_0 acts by integers then the action integrates to an action of \mathbb{T}_{rot} . Otherwise we get an action of some finite cover of \mathbb{T}_{rot}

