MORNING SESSION: OVERVIEW

SPEAKER: ANDRÉ HENRIQUES
TYPIST: EMILY PETERS

ABSTRACT. Notes from the "Conformal Field Theory and Operator Algebras workshop," August 2010, Oregon.

First, some algebra:

Main point in the beginning of Wasserman's paper: There is a strong analogy

Lie groups	Conformal field theory
(compact simple connected and simply-connected)	ie, conformal nets: strongly additive, split finite μ-index
$G = SU(N)$ classification of irreps by tableau of height at most N. (where deleting a column of height N doesn't change the rep)	$\widetilde{LG}_\ell$, $\ell > 0$ classification of irreps by tableau of height at most N and width at most ℓ

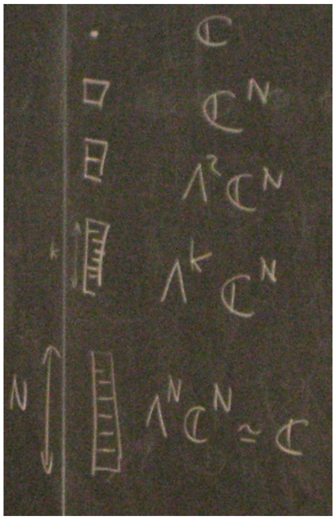
and we've been studying how to move from the left column to the right.

Definition. *split:* Given disjoint intervals I and J, consider the algebraic tensor product $\mathcal{A}(I) \otimes \mathcal{A}(J)$; the two different completions are equal.

Some examples of tableau and the corresponding representations.

Date: August 20, 2010.

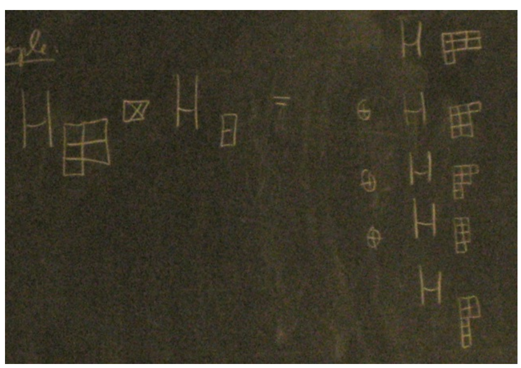
Available online at `http://math.mit.edu/~eep/CFTworkshop`. Please email `eep@math.mit.edu` with corrections and improvements!



Piere rule: $-\otimes V_{[k]}$ = add k boxes, no two in same row; sum over the corresponding irreducibles V_λ, with mutliplicity one.

The goal is to show the very same statement, with addition of an admissibility condition $f_1 - f_N \leq \ell$ on tableau:

$-\boxtimes H_{[k]}$= add k boxes, no two in same row; sum over the corresponding irreducibles H_λ, with mutliplicity one.

Example. 

Here's one difference between the two pictures: Free fermion rep has only one irrep while loop group has lots of them.

We have an action of $\tilde{LG}$ on Fock space.

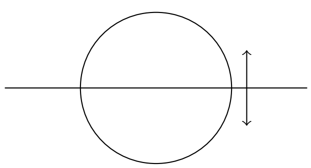
Notice that LG reps correspond $L\mathfrak{g}reps$, which correspond to $L^{pol}\mathfrak{g}$ reps, where we take the finite energy parts of the vector spaces and no longer have to worry about unbounded operators. This is very useful for classification results. For example, an $L^{pol}\mathfrak{g}$ rep gives us a rep of a dense subgroup of $\widetilde{LG}$; so classification of $L^{pol}\mathfrak{g}$ reps gives us an upper bound on the number of LG reps; then we use the action of $\widetilde{LG}$ on Fock space to actually construct these reps.

Virasoro algebra/ $Diff(S^1)$.

There is also an analytic side to this story:

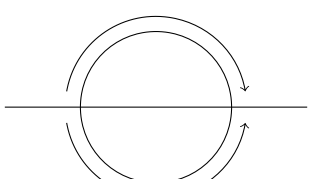
The main technical tool for studying (type III) von Neumann algebras is Tomita-Takesaki theory. One can build noncommutative L^p spaces such that $L^\infty(M) = M$, $L^1(M) = M_*$ (the predual), and $L^2(M)$ is some hilbert space – all of these are equalities as bimodules of M. Further if $M = \mathcal{A}(\frown)$ then $L^2(M)$ is the vacuum rep.

The *-operator on $L^2(M)$, typically called J, acts via



Is this breaking symmetry? No because we chose a special upper half circle already $M = \mathcal{A}(\frown)$.

We also have a modular flow $\Delta_\phi^{it}(\xi) = \phi^{it}\xi\phi^{-it}$ with $\phi = \langle \cdot \Omega, \Omega\rangle$; this acts via



–this is a geometric implementation of the modular flow (along the lines of the Bisognano-Wichman theorem).

We get factoriality because the modular group is ergodic, and that these factors are type III_1.

Thus: free fermions are a factor representation. Being a factor representation tells us that the vNa we get by completing $L_I(\tilde{G})_\ell$ acting on H_0 or $L_I(\tilde{G})_\ell$ acting on H_λ; a priori it's not clean at all why the two different completions $(L_I\tilde{(G)}_\ell)''$ should be the same; however, they both sit inside Fock space, and are both factor reps, so are canonically isomorphic. Thus we can talk about "the" level ℓ representation

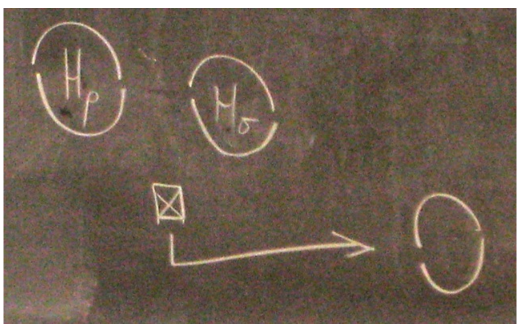
At this point we can talk about "the" conformal net associated to the loop group.

We can view these as bimodules for the algebra M, where we secretly identify the top and the bottom intervals via J;

(Finite index condition means) We get a braided rigid ribbon category. Many of these properties follow from there being only finitely many $L^{pol}\mathfrak{g}$ reps.

Question. Can you say something about fusion of conformal nets coming from loop groups?

Answer. Ah, so the question is, we have a rep of this style



but also need to know, say, that the left interval acts.

Solution: use the formalism Yoh used; $\rho, \sigma \in \mathrm{End}(\mathcal{A}(\frown))$ that act trivially near the boundary. "Localized endomorphisms."

Use that $H_\rho \simeq H_0$ equivariantly w.r.t. $\bigcirc$

We also had talks about primary fields:

Defined the beasts (Arturo classified them using algebraic approach, constructed them using fermions), Anatoly defined some function (four-point function) via power series, and we'll need to know that the value of the function at a certain point in non-zero. This is why we calculated the transport coefficients.