CLOSING REMARKS

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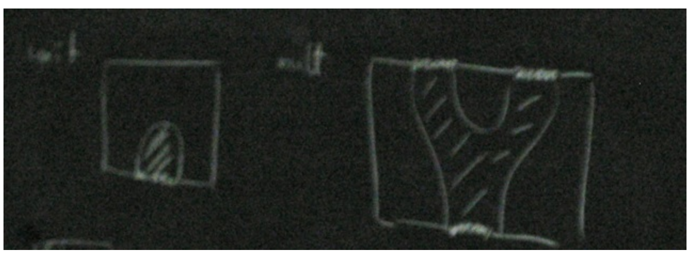
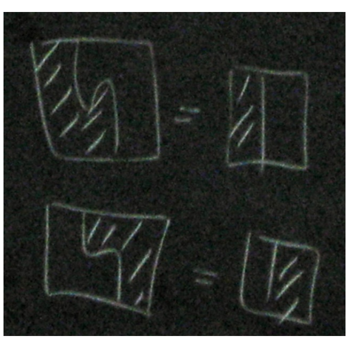
Abstract. Notes from the "Conformal Field Theory and Operator Algebras workshop," August 2010, Oregon.

diagrams for bimodules. Shaded corresponds to M, unshaded to N.

unit, multiplication:



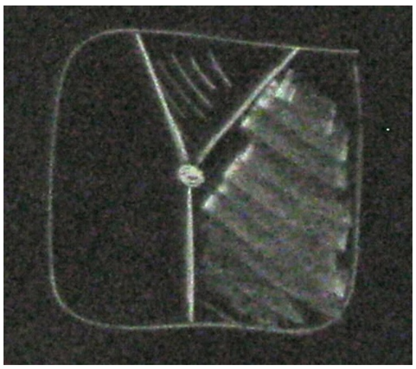
Advantage: use pictures to remember conditions. Disadvantage: we are secretly using other conditions, like finite index. For example, dualizability (defined pictorally below) relies on finite index:



Date: August 24, 2010.
Available online at http://math.mit.edu/∼eep/CFTworkshop. Please email eep@math.mit.edu with corrections and improvements!

bimodules correspond to codimension 1pictures; either points on lines or lines in the plane. morphisms between bimodules have codimension 2 – points in the plane.

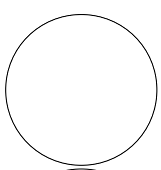
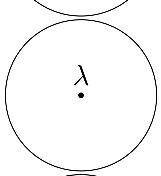
For example, here's a picture of an M, N bimodule fused wtih an N, P bimodule mapping to an M, P bimodule.

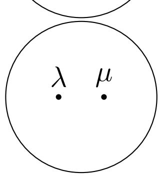


3-category:

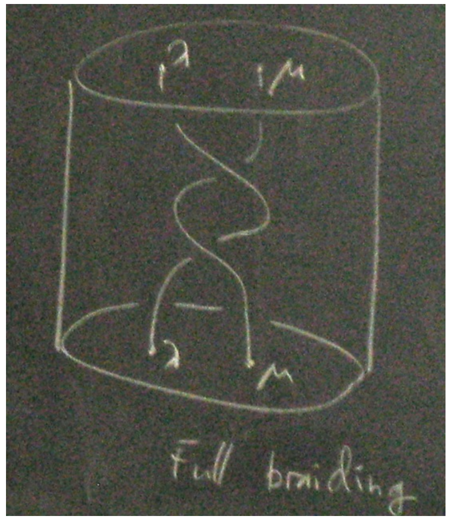
objects are codim-0
1-morphisms are codimention 1
2-morphisms are codimension 2
3-morphisms are codimension 3

Some examples of graphical notation:

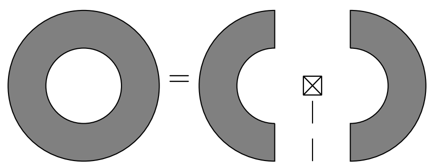
-  Corresponds to the vacuum hilbert space
- 

The hilbert space H_λ
- 

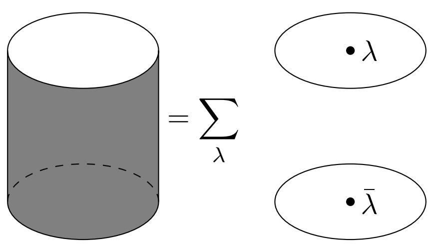
The hilbert space $H_\lambda \boxtimes H_\mu$.

- The braiding: 

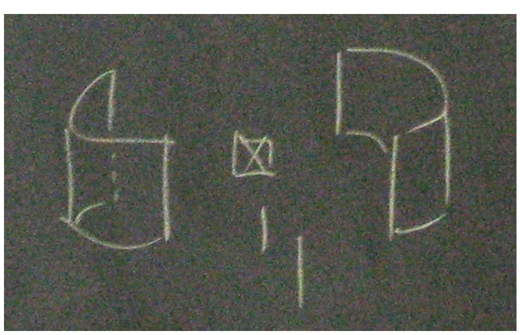
- Writing an annulus as the tensor (over a two-interval algebra) of two disks.

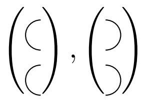


Lemma 0.1. *Neck cutting:*

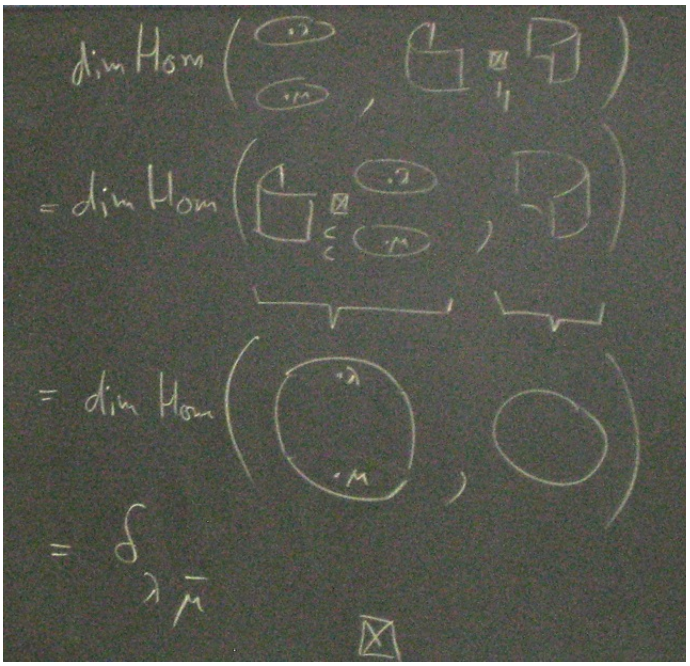


Proof. Finite μ-index implies



has finite dimension as a  bimodule.

we must compute the dimension of some Hom spaces. We use frobenius reciprocity (and secretly dualizability) and see



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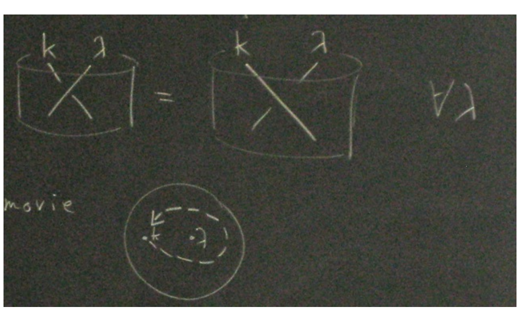
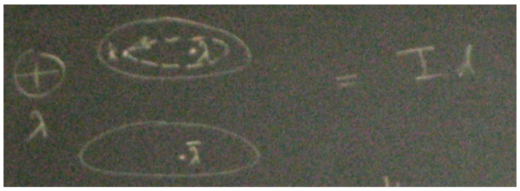
A diagrammatic argument for modularity:

Proof that $\mathrm{Rep}_f(\mathcal{A})$ *is modular:* Let k be a transparent object, ie an object such that the positive and negative braidings are equal for all λ, ie

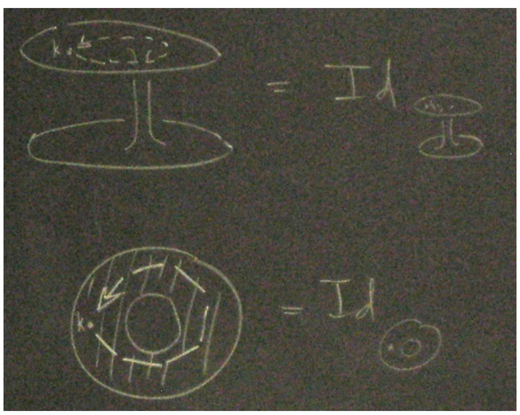


(note we can rewrite this by saying the full twist is the identity.) Think of $-z$ direction as time. As a movie, this is just k moving in a full circle once around λ.

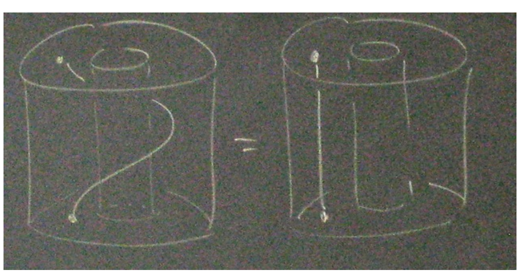
As a consequence, we get



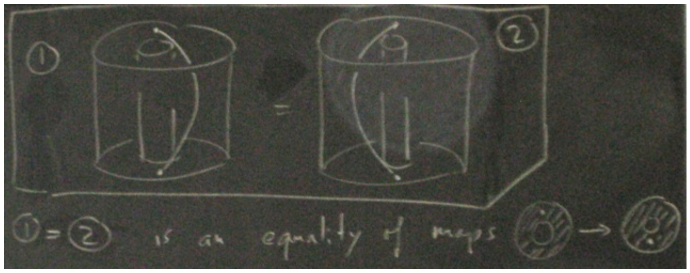
This can be rewritten using the neck cutting lemma; then we flatten this and get k taking a trip around this hole.

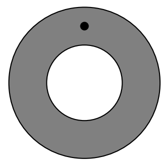
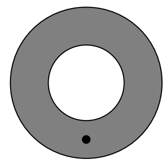


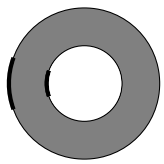
Now let's redraw this in 3D again:

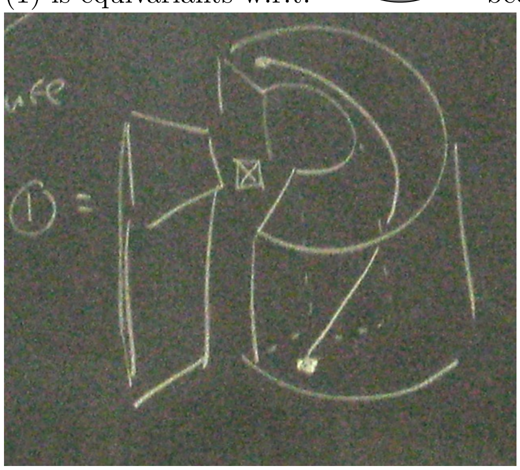


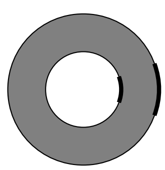
or

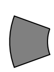
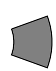


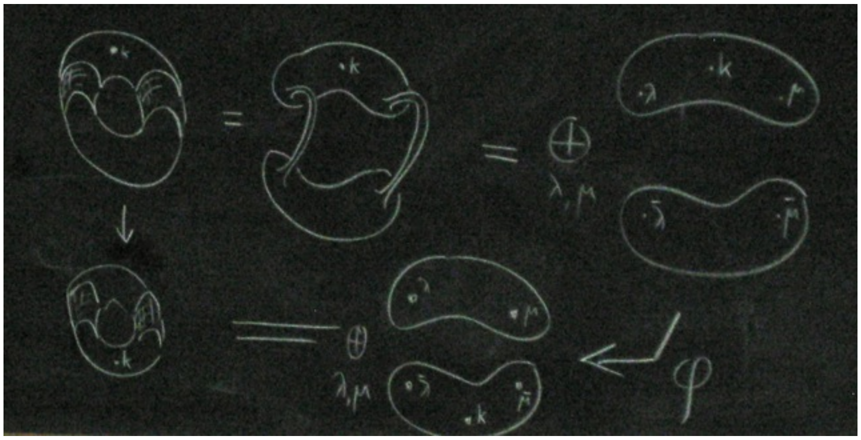
(1)=(2) is an equality of maps from  to 

(1) is equivariants w.r.t.  because



(2) is equivariant w.r.t.  for the same reason.

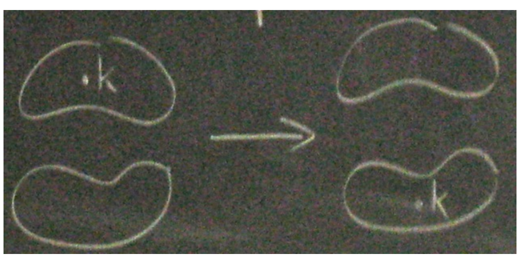
The map for (1)=(2) can therefore be fused with   to get a map



This looks like a torus, but is really two disks with tubes connecting them. (Audience member: shouldn't the tubes be going under the disks, in that second picture? André: yes, but I don't want to redraw them!)

The map ϕ_1 respects $\bigoplus_\lambda$; ϕ_2 respects $\bigoplus_\mu$.

Therefore $\phi = \phi_1 = \phi_2$ restricts to a map



For this map to be equivariant w.r.t. the action of the algebras, we need $k = 0$.

$\square$