QUESTIONS AND COMMENTSS

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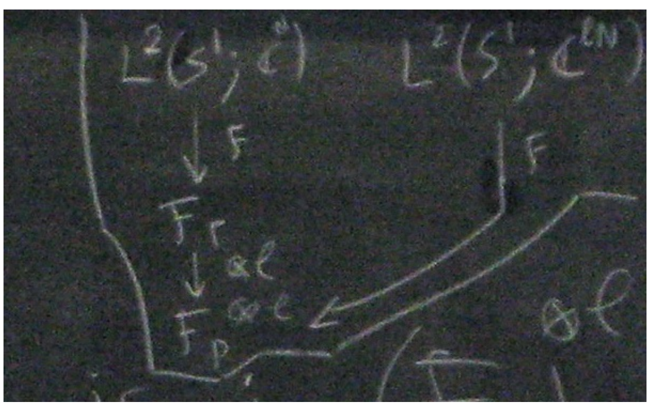
Abstract. Notes from the "Conformal Field Theory and Operator Algebras workshop," August 2010, Oregon.

Projective $SL_2(\mathbb{R})$ representations on various spaces. Some facts (they might seem contradictory).

$SL_2(\mathbb{R})$ acts on Fock space $\mathcal{F}_P^{\otimes \ell}$.

Every level ℓ LG representation appears in $\mathcal{F}_P^{\otimes \ell}$ (this is Wasserman's definition of level).



–this picture explains something about the appearance of N (of $SU(N)$) in this setting.

$SL_2(\mathbb{R})$ acts only *projectively* on H_λ.

Why isn't it an honest action? H_λ appears with huge multiplicity in $\mathcal{F}_P^{\otimes \ell}$ – the λ-isotypical component of $\mathcal{F}_P^{\otimes \ell}$ can be written as $H_\lambda \otimes$(Multiplicity space)

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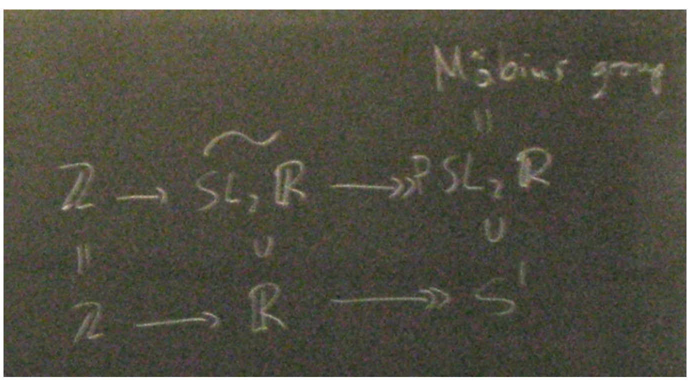
Available online at `http://math.mit.edu/~eep/CFTworkshop`. Please email `eep@math.mit.edu` with corrections and improvements!

– both components are projective reps of $SL_2(\mathbb{R})$, and when you tensor them together the cocycles cancel.

Question. What's up with the braiding on the category of conformal nets?

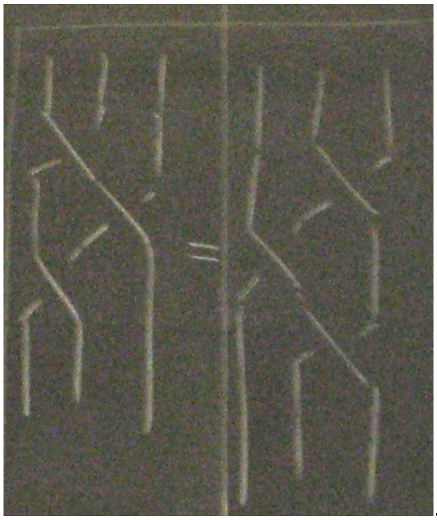
Universal central extension of $SL_2(\mathbb{R})$ (or $PSL_2(\mathbb{R})$) is $S\tilde{L_2}(\mathbb{R})$, extension of $SL_2(\mathbb{R})$ with fiber $\mathbb{Z}$.



Given a representation H_λ of a net (drawn in a circle), rotation by π introduces an isomorphism between $\mathcal{A}(I)$ and $\mathcal{A}(-I)$. Define H_λ^π to be H_λ with actions twisted by rotation by π. This is a new representation of the conformal net, which is isomorphic to the previous one, but not canonically. The half-twist θ– the element lifting π – is an LG-equivariant map $H_\lambda \to H_\lambda^p i$.

The braiding $\beta : H_\lambda \boxtimes H_\nu \to H_\nu \otimes H_\lambda$ is given by $\theta^{-1}_{H_\lambda \boxtimes H_\nu}(\theta_{H_\lambda} \circ \theta_{H_\nu})$.

Noah: is there any easy way to check this is actually a braiding, other that just satisfying the braid relations?

 , 

Question. Can you say something about how the primary fields appear in the Segal picture?

V is a rep of G, H are reps of LG

A primary field is an $\tilde{LG}_\ell \rtimes S^1$-equivariant map

$$C^\infty(S^1, V_k) \otimes H_j \to H_i, \tag{1}$$

which can be unbounded.

This encodes the same information as an $\tilde{LG}_\ell$-equivariant map

$$H_k \boxtimes H_J \to H_i, \tag{2}$$

even though the second is positive energy and the first isn't.

(2) is equivalent to $\mathrm{Hom}_{\tilde{L_I G}}(H_0, H_k) \otimes H_j \to H_i$.

I'm given $f \in C^\infty(S^1, V_k)$. What do I get from this function? From (1) I get $H_j \to H_i$. I also get (ah, now comes the circular reasoning) – if you already believe the equivalence between the $j = 0$, $i = k$ case of the correspondence, then $f \in C^\infty(S^1, V_k)$ induces a map $H_0 \to H_k$. Here we've used the obvious map $H_k \boxtimes H_0 \to H_k$.

So, a primary field takes a function f, produces a map $H_0 \to H_k$.

Okay, end circular reasoning. I also get, from f, using (2), an element in $\mathrm{Hom}(H_0, H_k)$.

Hmm ... can anyone say anything about the Segal picture?