Hello students:

(1) **#1:** This problem is turning out to be very difficult! In particular, there does not seem to be a clean, simple description of the language of the automaton $M_2$. If someone thinks they have such a description, please email me with your description. As for $M_1$, the language can be described as "the set of all strings such that each $a$ is preceded by an even number of $b$s." (This is what I mean by a clean, simple description of the language of an automaton.) The language of the cross-product machine $M_1 \times M_2$ is also hard to describe nicely. **Anyway, please delete this problem from the current assignment. I will try to find a better example to assign.**

(2) **p. 45, #20:** This problem looks hard but really is almost trivial. Show that $L^*$ and $L$ are almost exactly the same. In particular, $L^*$ contains just one more string than $L$ does. What is it?

(3) **p. 109, #6,8:** These problems illustrate closure properties of regular languages. We actually did #6 in class. For #8, express $cor(L_1, L_2)$ in terms of union and complement.

(4) **p. 121, #4(c):** Try the string $w = a^m b^n c^m$.

(5) **p. 121, #7:** This problem is challenging. (The Pumping Lemma is hard to apply directly to this problem.) To do this problem, show that the following language is not regular using the Pumping Lemma: $M = \{a^m b^n : n = k - 1\}$. When we do this, we can show that the given $L$ is not regular using the following line of reasoning. First notice that $L \cap \{a^m b^n : m \geq 0, n \geq 0\} = M$. Now if $L$ was regular, $L$ would be regular by closure properties. Clearly, the language $\{a^m b^n : m \geq 0, n \geq 0\}$ is regular. (It is simple to build a machine to accept this language!) Therefore, again by closure properties, $M = L \cap \{a^m b^n : m \geq 0, n \geq 0\}$ would have to be regular. But, $M$ is not regular! Therefore, $L$ cannot be regular completing the argument. So, all you have to do is show that $M$ is not regular. **This exercise shows how the closure properties of regular languages can be used indirectly to show that languages are regular or not regular.**

(6) **p. 121, #9(e):** Try using the string $w = a^m b^{2n}$. 