Hello students:

(1.) p. 121, #4(b): To do this problem, show that the following language is not regular using the Pumping Lemma: \( M = \{a^nb^k : k = n + l\} \). When we do this, we can show that the given \( L \) is not regular using the following line of reasoning. First notice that \( \mathcal{L} \cap \{a^mb^na^p : m \geq 0, n \geq 0, p \geq 0\} = M \). Now if \( L \) was regular, \( \mathcal{L} \) would be regular by closure properties. Show that the language \( \{a^mb^na^p : m \geq 0, n \geq 0, p \geq 0\} \) is regular. Therefore, again by closure properties, \( M = \mathcal{L} \cap \{a^mb^na^p : m \geq 0, n \geq 0, p \geq 0\} \) would have to be regular. But, \( M \) is not regular! Therefore, \( L \) cannot be regular completing the argument. So, all you have to do is:

1. Show that the language \( \{a^mb^na^p : m \geq 0, n \geq 0, p \geq 0\} \) is regular.
2. Show \( M \) is not regular by the Pumping Lemma.

(2.) p. 121, #9(c): Try using the string \( w = a^{m+1}b^{m+1} \).

(3.) p. 121, #9(d): Given an \( m \), let \( p \) be any prime number such that \( p \geq m \). Now try \( w = a^mb^p \). What about \( i \)? Try \( i = p + 1 \).

(4.) The alphabet here is \( \Sigma = \{a, b, c\} \). Note that the symbol \( c \) is acting like a marker symbol. Like problem #4(b), it is more convenient to apply closure properties to prove that \( L \) is not regular. Consider the language \( M = \{w_1cw_2 : w_1, w_2 \in \{a, b\}^* \text{ and } w_1 = w_2\} \). Clearly, \( M = \mathcal{L} \cap \{w_1cw_2 : w_1, w_2 \in \{a, b\}^* \} \). So, all you have to do is:

1. Show that the language \( \{w_1cw_2 : w_1, w_2 \in \{a, b\}^* \} \) is regular.
2. Show \( M \) is not regular by the Pumping Lemma.