Study Problems for Test 3
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Instructions. These study problems represent a sample of the types of problems that will be covered on Test 3. Some types of problems that may appear on the actual exam do not appear below and vice-versa.

1. Find the local linearization of the function \( f(x) = \sqrt{x} + 3 \) at the point \( x = 4 \), and use it to approximate the value of \( \sqrt{4.05} \).

2. The local linearization of what function around what point would be appropriate to approximate the value of \( \tan(44^\circ) \). What is this local linearization around the chosen point? You do not have to approximate \( \tan(44^\circ) \).

3. Differentiate the following functions.
   (a) \( f(t) = \frac{1}{2}t^6 - 3t^4 + t \)
   (b) \( g(x) = \frac{\sqrt{10}}{x^7} \)
   (c) \( h(x) = ae^x + \frac{b}{x} + \frac{c}{x^2} \) where \( a, b, \) and \( c \) are constants
   (d) \( u(x) = \sqrt{x^2 + 2\sqrt{x^3}} \)
   (e) \( v(x) = 5^{x^2+1} + 1 \)

4. (Challenge Problem) Find the equations of both lines through the point \((2, -3)\) that are tangent to the parabola \( y = x^2 + x \).

5. Show that the curve \( y = 6x^3 + 5x - 3 \) has no tangent line with slope 4.

6. Differentiate the following functions.
   (a) \( f(x) = (x^{-2} + x^{-3})(x^5 - 2x^2) \)
   (b) \( g(x) = \frac{1}{x^4 + x^2 + 1} \)
   (c) \( h(x) = x^{3/2}(x + ce^x) \) where \( c \) is a constant

7. Suppose that \( f(x) = e^x g(x) \) where \( g(0) = 2 \) and \( g'(0) = 5 \). Find \( f'(0) \).

8. If \( f(x) \) is a differentiable function, find an expression for \( g'(x) \) where

   \[
g(x) = \frac{1 + xf(x)}{\sqrt{x}}
   \]

9. A manufacturer of produces bolts of fabric with a fixed width. The quantity \( q \) of this fabric (measured in yards) that is sold is a function of the price (in dollars per yard), so we can write \( q = f(p) \). Then the total revenue earned with selling price \( p \) is \( R(p) = pf(p) \).
(a) What does it mean to say that $f(20) = 10,000$ and $f'(20) = -350$?
(b) Assuming the values in (a), compute $R'(20)$ and interpret the result.

10. Find the derivatives of the following functions.

(a) $f(x) = x \sin x$
(b) $g(x) = \frac{1+\sin x}{x+\cos x}$
(c) $f(x) = \csc x + e^x \cot x$

11. Find the equation of the tangent line to the curve $y = x \cos x$ at the point $(\pi, -\pi)$.

12. Differentiate the following functions.

(a) $f(x) = \tan(\sin x)$
(b) $g(x) = e^{\sqrt{x}}$
(c) $h(x) = (1 + 4x)^5(3 + x - x^2)^8$
(d) $u(x) = e^{-5x} \cos(3x)$
(e) $v(x) = \tan^2(3x)$
(f) $w(x) = \sqrt{x + \sqrt{x}}$
(g) $p(x) = 2^{3x^2}$
(h) $q(x) = \sin(\tan \sqrt{\sin x})$

13. Suppose that $h(x) = f(g(x))$ and $g(3) = 6, g'(3) = 4, f'(3) = 2, \text{ and } f'(6) = 7$. Find $h'(3)$.

14. Find the following derivatives.

(a) $f(x) = \ln(\cos x)$
(b) $g(x) = \ln(x + \sqrt{x^2 - 1})$
(c) $h(x) = \ln(e^{-x} + xe^{-x})$
(d) $u(x) = \log_2(1 - 3x)$
(e) $v(x) = \ln^2(1 + e^x)$
(f) $w(x) = \ln(\ln(\ln x))$