Instructions. These study problems represent a sample of the types of problems that will be covered on Test 3. Some types of problems that may appear on the actual exam do not appear below and vice-versa.

1. Consider the function \( f(x) = x^3 - 3x^2 + 1 \).

   (a) On what intervals is \( f(x) \) increasing? Decreasing?

   \[ \text{Answer: } (-\infty, 0), (2, \infty) \]

   (b) On what intervals is \( f(x) \) concave up? Concave down?

   \[ \text{Answer: } \text{Concave up on } (1, \infty), \text{concave down on } (-\infty, 1) \]

   (c) Find all the critical points of \( f(x) \).

   \[ \text{Answer: } x = 0, 2 \]

   (d) Find all the inflection points of \( f(x) \).

   \[ \text{Answer: } x = 1 \]

   (e) Determine which of the critical points are local minima and which are local maxima. What is the global minimum, and what is the global maximum?

   \[ \text{Answer: } x = 0 \text{ is a local maximum, } x = 2 \text{ is a local minimum; no global maximum or minimum} \]

   (f) Suppose the domain of \( f(x) \) is restricted to the interval \([-1/2, 4]\). What is the global minimum now, and what is the global maximum now?

   \[ \text{Answer: } x = 2 \text{ is a global minimum, } x = 4 \text{ is a global maximum} \]

2. Sketch the graph of a function \( y = f(x) \) that satisfies all the following conditions.

   (a) \( f'(0) = f'(2) = f'(4) = 0 \).

   (b) \( f'(x) > 0 \) if \( x < 0 \) or \( 2 < x < 4 \).

   (c) \( f'(x) < 0 \) if \( 0 < x < 2 \) or \( x > 4 \).

   (d) \( f''(x) > 0 \) if \( 1 < x < 3 \).
(e) \( f''(x) < 0 \) if \( x < 1 \) or \( x > 3 \).

3. Find a positive number such that the sum of the number and its reciprocal is as small as possible.

**Answer:** \( x = 1 \)

4. A farmer has 2400 ft. of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that can be fenced off that has the largest area?

**Answer:** 600 ft deep and 1200 ft wide

5. Find the point on the line \( y = 4x + 7 \) that is closest to the origin \((0, 0)\).

**Answer:** \((-28/17, 7/17)\)

6. At which points on the curve \( y = 1 + 40x^3 - 3x^5 \) does the tangent line have the largest slope?

**Answer:** \((-\sqrt{2}, 1 - 256\sqrt{2})\) and \((\sqrt{2}, 1 + 256\sqrt{2})\)
7. A certain line $L$ passes through the point $(3,5)$. If the area in the first quadrant bounded by the line $L$, the $x$–axis, and the $y$–axis is the smallest possible, find the equation of the line.

**Answer:** $y = -5/3x + 10$

8. A radar gun was used to record the speed of a runner during the first five seconds of a race. (See the table below.)

<table>
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<th>$t$ (sec)</th>
<th>$v$ (m/sec)</th>
<th>$t$ (sec)</th>
<th>$v$ (m/sec)</th>
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(a) Construct (and evaluate) a rectangle sum that uses five subintervals and left endpoints to estimate the distance traveled by the runner during the first five seconds. (Rectangle sums are more properly called Riemann sums.)

**Answer:** $\approx 38.34$ ft

(b) Construct (and evaluate) a rectangle sum that uses ten subintervals and right endpoints to estimate the distance traveled by the runner during the first five seconds.

**Answer:** $\approx 47.19$ ft