Math 162 Study Problems for Test 5
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Instructions. There are no special instructions for these study problems.

1. Determine the convergence or divergence of the following sequences. If the sequence converges, find the limit of the sequence.

   (a) \( a_n = \left(\frac{n+1}{2n}\right) \left(1 - \frac{1}{n}\right) \)

      Answer: Converges to \( \frac{1}{2} \)

   (b) \( a_n = \frac{\sin n}{n} \)

      Answer: Converges to 0

   (c) \( a_n = \frac{(-4)^n}{n!} \)

      Answer: Converges to 0

   (d) \( a_n = \ln n - \ln(n + 1) \)

      Answer: Converges to 0

   (e) \( a_n = \frac{n!}{n^n} \) (Hint: Compare with \( \frac{1}{n} \)).

2. Consider a sequence \( \{a_n\} \). Sometimes it is easier to determine the limit of the sequence \( \{a_n\} \) if we instead consider the sequence \( \{\ln(a_n)\} \). For example, suppose we can show that

   \[ \lim_{n \to \infty} \ln(a_n) = L \]

   Then, because of the continuity of the exponential function \( e^x \), we have that

   \[ \lim_{n \to \infty} e^{\ln(a_n)} = \lim_{n \to \infty} a_n = e^L \]

   Use this simple technique to determine the limits of the following sequences.

   (a) \( a_n = \left(\frac{1}{n}\right)^{1/n} \) for \( n > 1 \)

      Answer: Converges to \( e^{-1} \)

   (b) \( a_n = \sqrt[n]{n^2} \)

      Answer: Converges to 1

   (c) \( a_n = \left(1 + \frac{2}{n}\right)^n \)

      Answer: Converges to \( e^2 \)

3. Find the sum of the following series.
(a) \[ \sum_{n=3}^{\infty} \frac{1}{(2n-3)(2n-1)} \] (Hint: Use the technique of partial fractions to find an explicit formula for the \(n^{th}\) partial sum.)

Answer: \(1/6\)

(b) \[ \sum_{n=0}^{\infty} e^{-n} \]

Answer: \(\frac{e}{e-1}\)

4. Which of the series below converge absolutely, which converge conditionally, and which diverge?

(a) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} \]

Answer: Converges conditionally

(b) \[ \sum_{n=3}^{\infty} \frac{\ln n}{\ln(\ln n)} \]

Answer: Diverges

(c) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n \sqrt{n^2+1}} \]

Answer: Converges absolutely

(d) \[ \sum_{n=1}^{\infty} \frac{(-1)^n 3n^2}{n^3+1} \]

Answer: Converges conditionally

(e) \[ \sum_{n=1}^{\infty} \frac{n+1}{n!} \]

Answer: Converges absolutely

(f) \[ \sum_{n=1}^{\infty} \frac{(-1)^n (n^2+1)}{2n^2+n-1} \]

Answer: Diverges

(g) \[ \sum_{n=1}^{\infty} \frac{(-3)^n}{n!} \]

Answer: Converges absolutely

(h) \[ \sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n} \]

Answer: Converges absolutely

(i) \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}} \]

Answer: Converges absolutely
(j) \[ \sum_{n=2}^{\infty} \frac{1}{n \sqrt{n^2 - 1}} \]

**Answer:** Converges absolutely