The Firm in a Perfectly Competitive Economy:  
An Introduction to Duality Theory

Linear Programming can be used to explain the behavior of the firm operating in a perfectly competitive economy. In such an (idealized) economy, it is assumed that if the firm could make profits in excess of the value of its resources, then some other firm would enter the market with a lower price, thus tending to eliminate these excess profits.

The following assumptions will be made:

- The firm wishes to set activity levels for \( n \) activities (an ‘activity’ is either a product or a service) it plans to engage in. Let \( x_1, x_2, \ldots, x_n \) denote these activity levels.

- The \( n \) activities require the consumption of \( m \) resources. Let \( a_{ij} \) denote the amount of resource \( i \) that is consumed if the firm engages in one unit of activity \( j \), \( 1 \leq j \leq n, \quad 1 \leq i \leq m. \)

- The firm owns \( b_i \) units of resource \( i \), \( 1 \leq i \leq m. \) Resource \( i \) can be bought or sold in the market for \( v_i \) dollars.

- The firm can sell one unit of activity \( j \) in the market for \( c_j \) dollars, \( 1 \leq j \leq n. \)

- In general, the firm has control over its own activity levels \( x_j \), but the market is the mechanism that sets the prices \( v_i \) of the resources.

In addition to the above assumptions, it will be assumed that the market ‘acts’ like a competitor to the firm. That is, the market will try to minimize the profits of the firm since the firm’s profits are at the expense of other firms in the market. In a sense, the firm and the market are playing a game, the firm trying to set its activity levels so as to maximize its profits and the market trying to set the prices of the resources so as to minimize the profits of the firm. It will be shown that the maximum profit the firm can make is equal to the market’s evaluation of its initial endowment of resources. That is, the firm makes no excess profits.