Example 1  John and Mary are taking a math course in which there are only three grades: A, B, and C. The probability that John gets a B is 0.3, and the probability that Mary gets a B is 0.4. The probability that neither gets an A but at least one gets a B is 0.1. Find the probability that at least one of them gets a B but neither gets a C.

Solution 1  The sample space $S$ can be arranged in a $3 \times 3$ matrix.

\[
S = \begin{cases} 
  \text{JAMA, JAMB, JAMC}, \\
  \text{JBMA, JBMB, JBMC}, \\
  \text{JCMA, JCMB, JCMC} 
\end{cases}
\]

Let the events $D, E,$ and $F$ be defined as follows.

- $D$: John gets a B
- $E$: Mary gets a B
- $F$: at least one gets a B, but neither gets an A

Clearly, $D$ is the second row of the matrix, $E$ is the second column, and $F = \{JBMB, JBMC, JCMB\}$. Now consider the event we are interested in.

- $G$: at least one gets a B, but neither gets a C

Clearly, $G = \{JBMB, JAMB, JBMA\}$. Note that $D \cup E = F \cup G$. Therefore,

\[
\Pr(D \cup E) = \Pr(F \cup G)
\]

Using the formula for the union of two events, we obtain

\[
\Pr(D) + \Pr(E) - \Pr(D \cap E) = \Pr(F) + \Pr(G) - \Pr(F \cap G)
\]

and so since $\Pr(D \cap E) = \Pr(F \cap G)$ (since both sets are equal to $\{JBMB\}$), we can cancel to obtain

\[
\Pr(G) = \Pr(D) + \Pr(E) - \Pr(F) = 0.3 + 0.4 - 0.1 = 0.6.
\]