Quiz #3
Applied Calculus I – Math 131.009 – Fall 2012

Names: ______________________

Show your work for credit.

1. (2 pts) Exponential stuff.

(a) What annual percent decline in a bond is equivalent to a continuous rate of decline of 
0.0371?

(b) If \( B(3) = 3 \) and \( B(11) = 33 \), what is \( B(44) \)? Here time is measured in months, and bond value \( B \) is measured in hundreds of dollars. Give units in your answer.

\[
\begin{align*}
\text{1.} & \quad B(t) = P_0 \left(1 + r\right)^t = P_0 e^{0.0371 t} \quad \text{Solve for } r; \\
\text{2.} & \quad B(t) = B_0 e^{rt} \quad r = \frac{\ln\left(\frac{\text{终值}}{\text{初值}}\right)}{t} \end{align*}
\]

\[
\begin{align*}
\text{Solve for } r: & \quad \ln\left(\frac{33}{3}\right) = 0.0371t \\
& \quad 1 + r = e^{0.0371} \quad \Rightarrow \quad r = 0.0364
\end{align*}
\]

\[
\begin{align*}
\text{New Find } B_0: & \quad 33 = B_0 e^{3 \cdot 0.0371} \\
& \quad B_0 = 3 \cdot e^{-3 \cdot 0.0371} \quad \Rightarrow \quad B_0 = 2.36
\end{align*}
\]

\[
\begin{align*}
\text{New Find } B(44): & \quad B(44) = 2.36 e^{0.0371 \cdot 44} = 79.38
\end{align*}
\]

2. (1 pt) Transformation stuff. Let \( f(x) = x^2 + 1 \).

(a) Find a function \( g \) so that \( f(g(x)) = 9x^4 - 6x^2 + 2 \).

\[
\begin{align*}
\text{Fit is degree } 4, \text{ so must have composed with a degree } 2 \text{ function.} \\
(a^2 + b x + c)^2 + 1 &= a^2 x^2 + b^2 x^2 + c^2 + 2 abx + 2 acx^2 + 2 b c x + 1.
\end{align*}
\]

- Only degree 4 term is \( a^2 x^4 \), so \( a = 3 \).
- No degree 3 term in resulting \( f g \), so need \( 2ab = 0 \), so \( b = 0 \).
- \( \text{Term} x^2 \) \( 9x^4 - 6x^2 + 2 = 9x^4 + c^2 + 6c x^2 + 1 \)
- \( \text{Need} -6 = 6c \), so \( c = -1 \)
- That works, since \( 2 = \left(-1\right)^2 + 1 \).

\[
\begin{align*}
\text{g(x)} = 3x^2 - 1
\end{align*}
\]
3. (1 pt) Which of the following could best describe the graph in Fig. 1.25?

- (a) \( y = 3 \sin \left( \frac{x}{2} + \frac{\pi}{2} \right) \)
- (b) \( y = 3 \sin \left( 2x + \frac{\pi}{2} \right) \)
- (c) \( y = 3 \cos (2x) \)
- (d) \( y = 3 \cos \left( \frac{x}{2} \right) \)
- (e) \( y = 3 \sin (2x) \)
- (f) \( y = 3 \sin \left( \frac{x}{2} \right) \)

![Figure 1.25](image)

4. (1 pt) For the function shown in Figure 2.3, arrange the indicated numbers in increasing order.

- (a) 0
- (b) \( g'(-2) \approx 3 \)
- (c) \( g'(0) \approx 0 \)
- (d) \( g'(1) \approx -2 \)
- (e) \( g'(3) \approx -1 \)
- (f) \( g'(4) \approx 7 \)

![Figure 2.3](image)

\((c) < (d) < (g) < (b) < (f) < (e)\)