Answer Key: Practice Problems for Exam 1

Basic Linear Functions:

1. $y = 100 - 2x$
2. $y = -5.5 + 1.5x$
3. $y = -\dfrac{9}{4} - \dfrac{7}{4}x$
4. $y = -\frac{4}{3} + \frac{2}{3}x$

Applications of Linear Functions:

5.
   (a) At an altititude of 1800 meters, the temperature is 20°C.
   (b) $T = 29 - \dfrac{1}{200}A$
   (c) 19°C
   (d) 800 meters

6.
   (a) When the person begins the trip, they 300 miles from home.
   (b) The person is driving at a constant speed of 50 miles per hour away from home.
   (c) $D = 300 + 50t$

Applications to Economics:

7.
   (a) $C = 800,000 + 80q \qquad R = 200q$
   (b) The y-intercept of the cost is $800,000$. This represents the fixed costs of the company.
   (c) The marginal revenue is \$80. This means cost go up by \$80 each time one more unit is produced. The marginal revenue is \$200. This means each time one more unit is sold, revenue increases by \$200.
   (d) They need to sell $1,333,400$ or more units.
   (e)

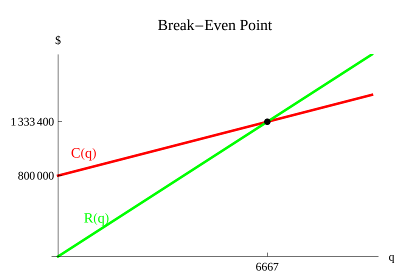

Figure 1: Cost and Revenue

8.

(a) $q = 310 - 20p$. When the item is free ($p = 0$), 310 units are demanded. Each time the price increases by \$1, 310 less units are demanded.

(b) $p = 15.5 - 0.05q$. If the item costs \$15.50, there will not be any demand. The slope means the same as in (a). Another way to phrase the meaning of the slope could be to say each time demand increases by 1 unit, the price must have dropped by \$0.05.

9.

(a) $q = 25000 - 20p$ represents the demand since the slope is negative. $q = 10p - 500$ represents the supply since the slope is positive.

(b) For every increase in price of \$1

- the quantity demanded goes down by 20 E-phones.
- the quantity supplied increases by 10 E-phones.

(c) The p-intercept of the demand means that if the price is \$1250, then there will be no demand. The q-intercept of the demand means if E-phones were free, then the demand would be $25,000$ E-phones.

(d) The p-intercept of supply means if the price for an E-phone is \$50, no manufacturers would be willing to supply any E-phones. The q-intercept of supply is -500. This cannot be explained meaningfully. If the price were zero than supply would be negative does not really apply to this situation.

(e) Equilibrium price is $p = \$850$, and equilibrium quantity is $q = 8,000$ E-phones.

(f)

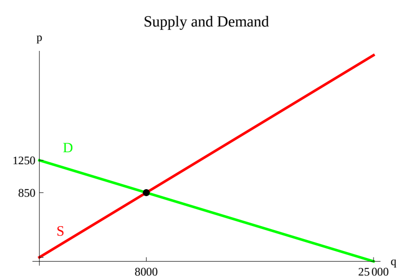

Figure 2: Supply and Demand

Basic Exponential Functions:

10. $y = 40 \cdot \left(\sqrt[3]{\frac{1}{5}}\right)^x \approx 40(0.5848)^x$

11. $y = 6.83(1.325)^x$ (rounding involved in my answer)

Applications of Exponential Functions and Logs:

12.

(a) $N(t) = 131(1.331)^t$

(b) The number of fatalities in 2005 was 131, at the number is growing by approximately 33.1% each year.

(c)

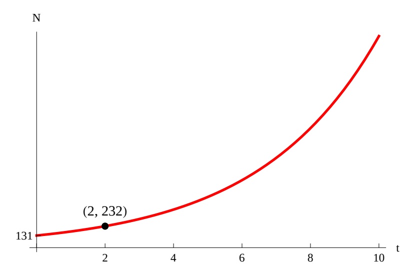

Figure 3: Coalition Military Fatalities in Afghanistan since 2005)

(d) In 2010 (when $t \approx 4.6$)

(e) The model above predicts $N(4) \approx 411$ which is considerably lower than the actual number reported for 2009. It makes sense our model predicts less since it is based on data prior to the increase of troops in Afghanistan.

(f) Approximately -28.6%

13.

(a) 1.26%

(b) 6.4 billion in 2004 and 6.9 billion in 2010.

(c) $P(5) = 6.8$. The world's population in 2009 was 6.8 billion.

(d) From 2004 to 2010, the world's population was growing on average by a rate of $83,300,000$ people per year.

14.

(a) $r \approx 0.364$ or approximately 36.4%.

(b) $N(t) = 2.5(1.364)^t$.

(c) 40.86 million internet hosts in Mexico.

(d) In 2043.

(e) From 2001 to 2008, the number of internet hosts in Mexico was growing on average by 2.79 million hosts each year.

15.

(a) $M(t) = 180.21e^{-0.02t}$
(b) Angola's infant mortality rate is predicted to be 163.06 in 2015.
(c) Not until 2045.
(d) Approximately -1.98%.

16. Two ways to think about this. You can compare either future or present values of each option. Both ways will lead to same conclusion.

- Present value of extending contract is simply \$200.
- Present value of not buying contract, and instead paying for repairs is \$255.14

Since the present value of extending the contract is less, that is the cheaper option.

- Future value of extending contract is \$246.73.
- Future value of not buying contract, and instead paying for repairs is \$314.76

Since the future value of extending the contract is less, that is the cheaper option.

Comparing Linear and Exponential Functions

17.

(a) $V(t) = 20,000 - 300t$
(b) $V(t) = 20,000(0.97)^t$
(c) $V(t) = 20,000e^{-0.03t}$

18.

(a) You are offered a \$80,000 starting salary and a fixed annual raise of \$2,000 each year.
(b) You are offered a \$70,000 starting salary and a fixed 4% raise each year.
(c) If you plan to stay at this job more than 8 years, take the salary in part (b). If you plan to work 8 years or less take the salary option in part (a).

Composition of Functions:

19.

(a) $f(2+h) = 3h^2 + 10h + 9$
(b) $f(2+h) - f(2) = 3h^2 + 10h$
(c) $\dfrac{f(2+h) - f(2)}{h} = 3h + 10$

20.

(a) $g(x) = 2x - 1$ and $f(x) = e^x$ (many other possibilities as well)
(b) $g(x) = \cos(x)$ and $f(x) = 5x - 7$ (many other possibilities as well)
(c) $g(x) = \sin(x)$ and $f(x) = x^2 + 4$ (many other possibilities as well)
(d) $g(x) = 3x - 1$ and $f(x) = \frac{1}{\sqrt{x}}$ (many other possibilities as well)

21.

(a) $f(g(4)) = 8$ (b) $g(f(8)) = 10$ (c) $f(f(6)) = 8$ (d) $g(g(10)) = 8$

Trigonometric Functions:

22.

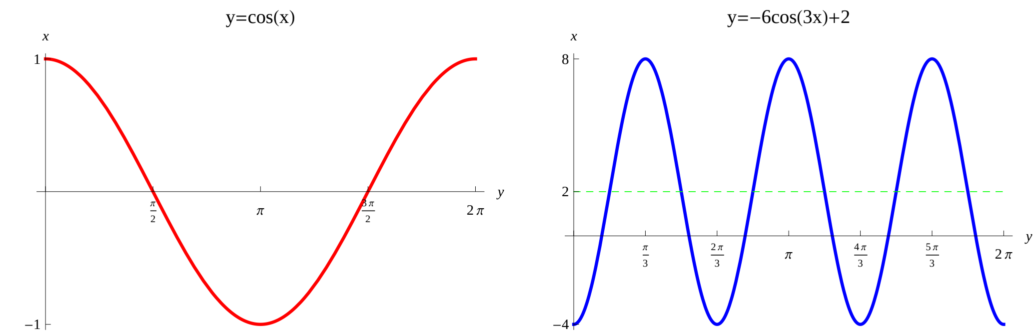

Figure 4:

23. $y = 5\sin\left(\frac{\pi}{4}x\right) - 3$

24.

(a)

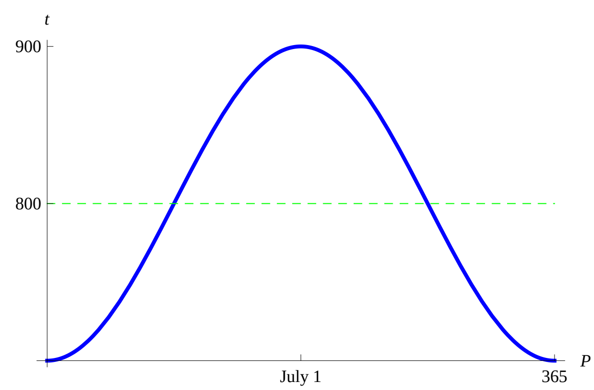

Figure 5: Population of Animals t days after January 1.

(b) Midline is $P = 800$. Amplitude is 100. Period is 365 days.

(c) $P = -100\cos\left(\frac{2\pi}{365}t\right) + 800$

Instantaneous Rate of Change and the Derivative:

25.

(a) Calculating using only a small interval after $t = 2$ yields the approximation

$$f'(2) \approx \frac{f(4) - f(2)}{4 - 2} = \frac{24 - 18}{4 - 2} = 3$$

Improving this approximation by taking into account the ARC over the prior interval $x = 0$ to $x = 2$ (the ARC over $x = 0$ to $x = 2$ is 4) would yield $f'(2) \approx 3.5$ **On the exam either approximation will be accepted.**

(b) $f'(x) > 0$ when $0 \leq x \leq 4$ and $f'(x) < 0$ when $6 \leq x \leq 12$.

26.

(a) 24 meters per second.

(b) Approximate the instananeous velocity of the particle at $t = 1$ by computing the average rate of change from $t = 1$ to $t = 1 + h$ for

(a) 18 meters per second (b) 12.6 meters per second (c) 12.06 meters per second.

27.

(a) $g'(x) > 0$ at A, B, and D. $g'(x) < 0$ at C and F. $g'(x) = 0$ at E.

(b) Greatest at B and least at F.

28.

(a) x_1, x_2, and x_3

(b) x_1, x_2, and x_5

(c) Same question as (b) phrased differently. Thus, x_1, x_2, and x_5

(d) x_1 (slope is becoming more positive after x_1), x_4, (slope is becoming less negative after x_4), and x_5 (slope is becoming more positive after x_5).

(e) Same question as (b) and (c) phrased differently. Thus, x_1, x_2, and x_5

(f) Same question as (d). Thus at x_1, x_4, and x_5.

29.

(a)

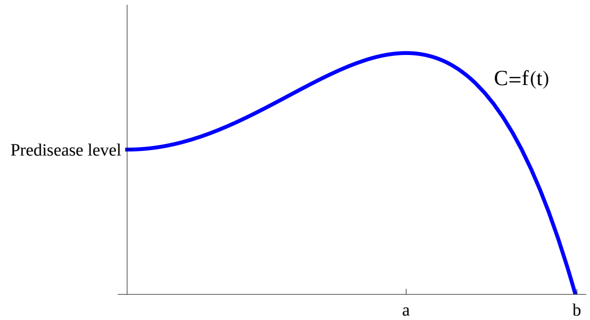

Figure 6: Enzyme Levels in a Patient with Liver Disease

(b) $f'(t) > 0$ for $0 < t < a$ and $f'(t) < 0$ for $a < t < b$.

(c) $f'(t)$ represents the rate at which enzyme levels are going up or down.

30.

(a) ARC $= 25$ megawatts per year. This means that from 1994 to 1998, the world solar energy output on average was increasing at a rate of 25 megawatts per year.

(b) $f'(4) \approx 18$ (your approx may vary slightly). This means that in 1994 the world solar energy output was increasing at a rate of 18 megawatts per year.

31.

(a) $h(6000) = 8000$. When a climber on Mount Everest is 6000 meters from the start of the trail, they are at an elevation of 8000 meters above sea level.

(b) When a climber is 6000 meters from the start of the trial, the trail's elevation is increasing by 0.5 meters for every meter further they hike.

(c) $h(6003) \approx h(6000) + h'(6000) \cdot \delta h = 8000 + 0.5(3) = 8001.5$ meters.

32.

(a) $f'(t)$ is negative since the temperature of the coffee will decrease.

(b) Degrees Celsius per minute. $f'(20)$ is the rate at which the temperature of the coffee is decreasing 20 minutes after the cup is placed on the counter.

33. Sketch derivatives for each of the following functions:

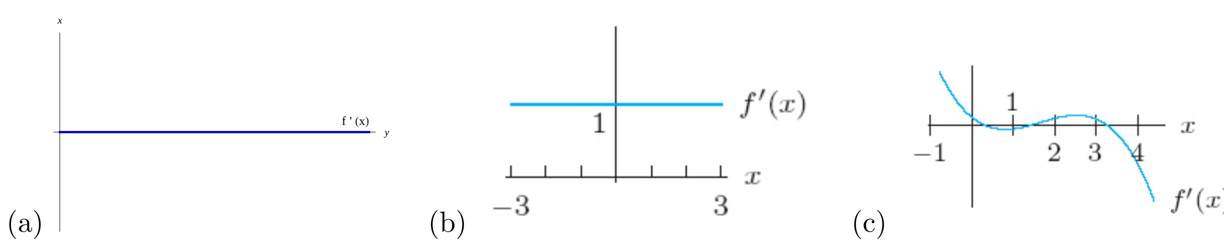

Figure 7: