Practice Problems Exam 3

1. Consider the function \( f(x) = x^2 + e^{2x} \) over the interval \(-1 \leq x \leq 3\), and answer the following questions.

   (a) Find all critical points in this interval, and identify each as a local max, a local min, or neither a local max or min.

   (b) Determine where on this interval the global maxima and minima occur (if ever).

2. Over what interval(s) is \( f(x) = 2x^3 - 12x \) decreasing and concave up? Give the EXACT interval(s) show the calculus required to determine your answer.

3. Sketch a possible graph of a function \( y = f(x) \) whose first and second derivatives satisfy the conditions below:

\[
\begin{array}{cccccccc}
\text{f''(x)} & + & + & + & + & + & + & + \\
\text{x} & -2 & 0 & 3 & 1 & 3 & 0 & 0 \\
\text{f'(x)} & - & - & - & - & - & + & + \\
\end{array}
\]

4. Given the curve \( y = ax^3 + bx^2 + cx + d \), with \( a \neq 0 \), find the relation between the constants \( a, b, \) and \( c \) that will ensure that the curve has no local maxima or local minima.

5. Suppose \( f(x) \) has a continuous derivative whose (the derivative’s) values are given in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) )</td>
<td>-2</td>
<td>-1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>-1</td>
<td>-3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

   (a) Give an \( x \) value where a local minimum could possibly occur?

   (b) Give an \( x \) value where a local maximum could possibly occur?

   (c) Give an \( x \) value where an inflection point could possibly occur?

6. Consider the graph below:

   (a) If you are told the graph above is of \( y = f(x) \), estimate where \( f(x) \) has inflection points.

   (b) If you are told the graph above is of \( y = f'(x) \), estimate where \( f(x) \) has inflection points.

   (c) If you are told the graph above is of \( y = f''(x) \), estimate where \( f(x) \) has inflection points.
7. Given the graph of \( y = f(x) \) below, locate all of the global maxima and minima.

![Graph of y = f(x)](image)

8. A brick is heated in an oven and taken out to cool off after a certain time. The temperature \( T \) of the brick at any time \( t \) is given by \( T = 103e^{-(t-1)^2} \) for \( t \geq 0 \), with \( T \) in degrees Celsius and \( t \) in minutes.
   
   (a) What is the initial temperature of the brick?
   (b) What is the maximum temperature the brick reaches? When does this occur? Do not solve this by graphing, solve this using calculus!

9. The number of plants in a terrarium is given by the function, \( P(x) = -2x^2 + 6x + 11 \), where \( x \) is the number of mg of plant food added to the terrarium. What is the amount of plant food the produces the highest number of plants? Include units and round to the nearest hundredth). What is the maximum number of plants when this amount of food is used?

10. Given the following table of production quantities with their corresponding marginal revenue and marginal cost.

<table>
<thead>
<tr>
<th>( q )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( MR )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( MC )</td>
<td>40</td>
<td>75</td>
<td>100</td>
<td>120</td>
<td>150</td>
<td>190</td>
</tr>
</tbody>
</table>

   (a) What is the price per unit?
   (b) What level of production should they choose to maximize profit?

11. A factory produces a product that sells for $11. They currently produce 2400 items per month, at an average cost of $5 per item. The marginal cost at this level is $4.

   (a) What is the profit at this production level?
   (b) Would increasing production increase or decrease profit?
   (c) Would increasing production increase or decrease average cost?

12. The average cost per item to produce \( q \) items is given by

   \[ a(q) = -0.02q^2 + 4q + 1. \]

   At what level of production will average costs be minimized?

13. Given the cost function \( C(q) = 3000 + 40q + 0.003q^2 \) and the demand function \( p = 120 - 0.035q \), find the value of \( q \) (to the nearest whole number) for which
(a) revenue is maximized.
(b) profit is maximized.

14. There is only one barber in a small town. Would you expect the price of a haircut to be elastic or inelastic? Explain.

15. If a good is elastic, will a price increase result in an increase or decrease of revenue? Explain how you determined your answer.

16. The demand curve for a product is given by \( q = 2400 - 5p^2 \).

   (a) Write revenue as a function of price and take its derivative to find the price (to the nearest cent) that will maximize revenue.

   (b) Find the elasticity (to two decimal places) at the price you found in part (a).

17. The drug concentration curve for a drug after \( t \) hours is given by \( C(t) = 11.5te^{-0.3t} \), where \( C \) denotes the concentration in ng/ml.

   (a) What is the initial concentration level?

   (b) When does the concentration reaches its maximum level, and what is the maximum level?

   (c) The minimum effective concentration is 10 ng/ml. Over what time period is the drug effective?

   (d) Sketch a graph of the concentration curve, labeling the points you found above.

18. Three different surge functions are graphed below:

   (a) Which graph corresponds to equation which has the largest \( b \) value?

   (b) Which graphs appear to have the same \( b \) values?

19. Does the maximum output value of the surge function increase, decrease, or stay the same when

   (a) \( b \) is increased and \( a \) is held constant.

   (b) \( a \) is increased and \( b \) is held constant.

20. A disease begins to spread in a small town. The number of people infected is modeled by the equation

\[
N(t) = \frac{300}{1 + 282e^{-0.07t}},
\]

where \( t \) is measured in days since the disease is first detected.

   (a) How many people initially have the disease at time \( t = 0 \)?

   (b) How many people eventually get the disease?
(c) Approximately when is the disease spreading fastest?

21. In Wilson corners, population 3500, a rumor spreads according to the logistic model. If 6 people initially know the rumor at 4 PM and 120 people have heard it by 5 PM, how many people will have heard the rumor by 6 PM (to the nearest person)?