Show your work for credit.

1. A soccer player is standing on the sideline, at a distance \( x \) feet from the goal line. The near edge of the goal is 30’ from the corner, and goal mouth is 20’ wide.

\[
\theta = \theta_2 - \theta_1
\]

(a) Write down a formula for the perceived size of the goal (i.e., viewing angle \( \theta \)), as a function of distance \( x \).

\[
\tan \theta_2 = \frac{30}{x} \quad \Rightarrow \quad \theta_2 = \arctan\left(\frac{30}{x}\right)
\]

\[
\tan \theta_1 = \frac{20}{x} \quad \Rightarrow \quad \theta_1 = \arctan\left(\frac{20}{x}\right)
\]

\[
\theta(x) = \arctan\left(\frac{30}{x}\right) - \arctan\left(\frac{20}{x}\right)
\]

(b) Using your calculator, sketch a graph \( \theta(x) \) on the interval \([0, 150] \).

(c) What is the sign of \( \theta'(35) \)? of \( \theta'(45) \)?

\( \theta'(35) : \text{positive} \quad \theta'(45) : \text{negative} \)

(d) Estimate the distance \( d \) that makes the goal mouth appear largest.

\( 38.5 \)

(e) Support your answer by estimating \( \theta'(d) \).

\[
\text{use } h = 0.001: \quad \frac{\theta(38.501) - \theta(38.5)}{0.001} = 3.73 \times 10^{-5} \quad \text{... so, yeah, basically zero.}\]
2. In this exercise, we take multiple derivatives, \( f \rightarrow f' \rightarrow f'' \rightarrow f''' \). Record which is which to complete the key (legend) at left.
3. You are hiking the Sauk Point Trail at Devils Lake near Madison, Wisconsin. Let $f(x)$ record the elevation of the trail, in feet, $x$ miles from the trailhead.

(a) What are the units of $f'(x)$?

\[
\frac{\text{feet}}{\text{mile}}
\]

(b) In practical terms, what does it mean if $f(2) = 600$ and $f'(2) = 3400$?

(If it stayed that steep over the next mile, you would be at 4000 ft. at mile 3. This is Wisconsin, not Colorado, so that won't happen.)

4. A function $P$ has $P(7) = 20$ and $P'(7) = 2$, and $P''(x) < 0$ for $x \geq 7$. Which of the following are possible values for $P(9)$ (and which are impossible)?

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If slope stayed constantly 2, then $P(9)$ would be 24. But we are told that $P$ is concave down.

Now suppose, instead, that $P''(x) = 0$ for $x \geq 7$. Which of the following are possible values for $P(9)$ (and which are impossible)?

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