Classwork for 3.1
Applied Calculus I – Math 131.00(7,8) – Fall 2013

Names: ____________________________ Solution Key__________________________

Show your work for credit.

1. We consider the parabola \( y = f(x) = \frac{x^2}{3} + 3 \).

   (a) Find a formula for the slope of the tangent lines to \( f \) at the points \( x = a \) and \( x = -a \).

   \[
   f'(x) = \frac{2}{3}x
   \]

   \[
   f'(a) = m = \frac{2}{3}a = \frac{2a}{3} \quad \quad \quad f'(-a) = M = \frac{2}{3}(-a) = -\frac{2a}{3}
   \]

   (b) Determine \( a \) so that these two tangent lines intersect perpendicularly. Sketch a picture.

   \[
   \text{Need} \quad m = \frac{-1}{M} \quad \text{so} \quad \frac{2a}{3} = \frac{-1}{\frac{2a}{3}} = \frac{-1}{1} \quad \frac{a}{2a} = \frac{3}{2a}.
   \]

   \[
   \text{i.e.,} \quad \frac{2a}{3} = \frac{3}{2a} \quad \text{or} \quad 4a^2 = 9 \quad \Rightarrow \quad a = \sqrt{\frac{9}{4}} = \frac{3}{2}
   \]

   (c) Find an equation for one of the tangent lines above.

   \[
   \text{the left one:} \quad y - y_1 = m(x - x_1)
   \]

   \[
   y - \left[ \frac{a^2}{3} + 3 \right] = \left[ \frac{2a}{3} \right] \left( x - \left[ a \right] \right)
   \]

   \[
   y = \left( \frac{1}{3} \left( \frac{3}{2} \right)^2 + 3 \right) + \left( \frac{2}{3} \left( \frac{3}{2} \right) \right) \left( x - \frac{3}{2} \right)
   \]
2. Let \( f(t) \) and \( g(t) \) give, respectively, the amount of water (in acre-feet) in two different reservoirs on day \( t \). Suppose that \( f(0) = 2000 \), \( g(0) = 1500 \), \( f'(0) = 11 \), and that \( g'(0) = 8 \). Let \( h(t) = f(t) - 2g(t) \).

(a) Evaluate \( h(0) \) and \( h'(0) \). Don’t forget units.

(b) What do these quantities tell you about the region’s water supply?

(c) Assume \( h' \) is constant for \( 0 \leq t \leq 800 \). Then \( h \) will have a zero in this interval. Find it.

(d) Now assume \( h \) is concave for \( 0 \leq t \leq 800 \). Does \( h' \) have any zeros? Does \( h \) have any zeros? Sketch a few possible scenarios.
3. Find an appropriate window in which to view the function \( F(h) = \frac{\sin(4 + 2h) - \sin(4)}{h} \). Use that window to estimate the value of

\[
\lim_{h \to 0} F(h).
\]

Use your answer to lend credence to the claim that \( \frac{d}{dx} (\sin 2x) \bigg|_{x=2} = 2 \cos(4) \).

I used

| \( x_{\text{min}} \) | -2.5 |
| \( x_{\text{max}} \) | 2.5 |
| \( y_{\text{min}} \) | -1.5 |
| \( y_{\text{max}} \) | -1.5 |

\[
\frac{d}{dx} (\sin 2x) \bigg|_{x=2} = \lim_{h \to 0} \frac{\sin(2(2+h)) - \sin(2)}{h} = \lim_{h \to 0} F(h).
\]

It looks like

\[
\lim_{h \to 0} F(h) \approx -1.30
\]

you tell me that the answer should be

\[
2 \cos(4) \approx -1.307
\]

4. Compute the following:

(a) \( \frac{d}{dx} \left( 2x^7 - \sqrt[3]{x} + \frac{x+1}{x^5} \right) - x \)

\[
= \frac{d}{dx} \left( 2x^7 - \frac{1}{3}x^{\frac{1}{3}} + x^{-4} + x^{-5} \right) - x
\]

\[
= 14x^6 - \frac{1}{9}x^{-\frac{2}{3}} - 4x^{-5} - 5x^{-6} - x
\]

(b) \( f'(x) \), if \( f(x) = (x^2 + x)(x^3 - 2) \)

\[
f' = 5x^4 - 4x + 4x^3 - 2
\]

(c) \( \frac{d^2}{dx^2} \left( x^6 + \frac{x^4}{2} - 2x \right) = \frac{d}{dx} \left( \frac{d}{dx} \left( x^6 + \frac{x^4}{2} - 2x \right) \right) \)

\[
= \frac{d}{dx} \left( 6x^5 + \frac{4x^3}{2} - 2 \right) = 30x^4 + 6x^2
\]

(d) \( f'(x) \), if \( f(x) = (x^2 + x)^3 \)

\[
f' = 6x^5 + 10x^4 + 8x^3 + 6x^4 + 3x^2
\]