1. If $p(x)$ is a density function for annual income (in thousands of dollars) of an individual working in the United States and $P(x)$ is the corresponding cumulative distribution function of $p(x)$, explain in everyday language to a non-calculus student (or instructor) the meaning of the following:
   
   (a) $p(40) = 0.012$
   (b) $P(75) = 0.913$.

2. What key features must the graph of a density function satisfy? Explain in practical terms why each of these features must be reflected in the graph.

3. What key features must the graph of a cumulative density function satisfy? Explain in practical terms why each of these features must be reflected in the graph.

4. A density function for the age of people enrolled in a class is given in the following figure.

   ![Density Function Graph](image)

   (a) Find the value of $c$.
   (b) What percent is over 20 years old?
   (c) Is the mean age of the class greater than, less than, or equal to the median age of the class? Explain how you determined your answer.

5. A density function for the lifetime of a certain type of frog is shown in the following figure.

   ![Lifetime Function Graph](image)

   (a) What is the most likely lifetime for a frog of this type?
   (b) Approximately what is the mean lifetime for a frog of this type?
   (c) Approximately what is the median lifetime for a frog of this type?
6. The density function \( f(x) \) shown below describes the probability that a computer circuit board will cost a manufacturer more than a certain number of dollars to produce. In this case, the cost of the circuit board, \( x \), is measured in thousands of dollars.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
x (thousand $) & 0 & 2 & 4 & 6 & 8 & 10 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
p(x) & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon \\
\hline
\end{array}
\]

(a) What is the probability that the circuit board will cost more than $10 thousand to produce?
(b) What is the value of \( \epsilon \)?
(c) What is the probability that the circuit board will cost between $4 thousand and $6 thousand to produce?
(d) What is the median cost to produce a circuit board?
(e) What is the mean cost to produce a circuit board?
(f) Sketch the corresponding cumulative distribution function.

7. The cumulative distribution function for the time to complete a step on an assembly line is given in the table below.

<table>
<thead>
<tr>
<th>( t ) (min)</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(t) )</td>
<td>0</td>
<td>0.03</td>
<td>0.15</td>
<td>0.21</td>
<td>0.54</td>
<td>0.78</td>
<td>0.86</td>
<td>0.97</td>
</tr>
</tbody>
</table>

(a) What percent of the steps takes between 10 and 14 minutes to complete?
(b) Approximately what is the median number of minutes it takes to complete a step?

8. The life expectancy of a bug can be approximated by the density function \( p(t) = 0.3e^{-0.3t} \), where \( t \) is time in days.

(a) What is the probability that a bug will live between 3 and 5 days?
(b) What is the median number of days a bug lives?
(c) What is the mean number of days a bug lives?

9. The annual rainfall (in inches) for a desert city is approximately normally distributed with mean \( \mu = 6 \) inches and a standard deviation of \( \sigma = 1 \). What is the probability that the annual rainfall in a given year will be between 5 and 8 inches?
10. The number of lift ticket sold, \( S \), at a ski resort are a function of the price, \( p \), of a ticket and the current number, \( n \), of inches of snow on the mountain. Thus, \( S = f(p, n) \).

(a) Do you expect \( f \) to be an increasing or a decreasing function of \( p \)?
(b) Do you expect \( f \) to be an increasing or a decreasing function of \( n \)?

11. A certain piece of electronic surveying equipment is designed to operate in temperatures ranging from 0°C to 30°C. Its performance index, \( p(t, h) \), measured on a scale from 0 to 1, depends on both the temperature \( t \) and the percent humidity \( h \) of its surrounding environment. Values of the function \( p = f(t, h) \) are given in the following table. (The higher the value of \( p \), the better the performance.)

<table>
<thead>
<tr>
<th>( t ) (°C)</th>
<th>0</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.38</td>
<td>0.46</td>
<td>0.43</td>
<td>0.28</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.65</td>
<td>0.79</td>
<td>0.73</td>
<td>0.47</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>0.81</td>
<td>0.99</td>
<td>0.91</td>
<td>0.59</td>
<td>0.02</td>
</tr>
<tr>
<td>30</td>
<td>0.71</td>
<td>0.87</td>
<td>0.81</td>
<td>0.52</td>
<td>0.01</td>
</tr>
</tbody>
</table>

(a) If the equipment has a performance index of 0.81 and the humidity is 50%, what is the outside temperature?
(b) Approximate \( \frac{\partial p}{\partial t} \bigg|_{(10,25)} \).
(c) Using everyday language, explain the meaning of your answer to (b) in practical terms.

12. The fuel cost \( C \) (in dollars) for a 3000 mile trip depends on the price \( p \) (in dollars) per gallon of gas and fuel economy \( m \) (in miles per gallon) of the car according to the formula below,

\[
C = f(p, m) = \frac{3000p}{m}.
\]

(a) On the same set of axes, sketch a graph of \( C \) as a function of \( p \) with fixed mileage rates of 10, 20, 30, and 40 mpg.
(b) On the same set of axes, sketch a graph of \( C \) as a function of \( m \) with fixed price per gallon $2, $3, $4, and $5.
(c) Calculate \( f(3, 30) \) and explain the meaning of this value in practical terms.
(d) Calculate \( f_p(3, 30) \) and explain the meaning of this value in practical terms.
(e) Calculate \( f_m(3, 30) \) and explain the meaning of this value in practical terms.

13. Sketch a contour diagram for \( z = f(x, y) = x^2 - 2y \) including contours at \( z = -5, 0, 5, 10, 15, \) and 20.
14. The heat index $H$ tells you how hot it feels as a result of the combination of temperature and humidity. The figure below gives a contour diagram for the heat index $H$ as a function of temperature $T$ (in °F) and humidity $p$ (as a %). Heat exhaustion is likely to occur when the heat index is 105 or higher. Note that the values of each contour line are labeled across the top of the diagram below.

(a) If the temperature is 100°F, at about what humidity level does heat exhaustion become likely?
(b) Approximate and explain the practical meaning of $H_p(95, 40)$.
(c) Approximate and explain the practical meaning of $H_T(95, 40)$.

15. An airline’s revenue, $R = f(x, y)$, is a function of the number of full price tickets, $x$, and the number of discount tickets, $y$, sold. When 300 full price and 600 discount tickets are sold, $R = 225,000$. If $f_x(300, 600) = 350$ and $f_y(300, 600) = 200$, approximate the revenue when 298 full price and 605 discount tickets are sold.

16. If $f(x, y) = x^2 e^{xy}$, find formulas for $f_x(x, y)$ and $f_y(x, y)$.

17. If $f(x, y) = \frac{x^2}{y}$, find formulas for $f_{xx}(x, y)$, $f_{yy}(x, y)$, $f_{xy}(x, y)$, and $f_{yx}(x, y)$.

18. Find and classify all critical points of $f(x, y) = x^3 - 3x + y^2$. 


19. The contour diagram for \( f(x, y) \) is provided below.

(a) Which (if any) of the points A-E have \( f_x(x, y) = 0 \)?
(b) Which (if any) of the points A-E have \( f_y(x, y) = 0 \)?
(c) Find and classify all critical points of \( f(x, y) \).

20. Find values for the constants \( A \) and \( B \) so that \( f(x, y) = Ax^2 + Bxy + y^2 \) has a critical point at \((1, 3)\).