Your final exam is scheduled for **Saturday, December 17**, at 9:00 a.m. in our usual room.

You may bring an $8\frac{1}{2} \times 11$ inch, one-sided, sheet of paper with formulas and definitions to the final exam. No other books or notes will be allowed.

Calculators will be allowed.

The exam will last two hours.

The exam will have between fifteenth (15) and twentieth (20) questions, covering Chapters 5–10.

Most questions will be similar in difficulty to the ones below (if not outright similar).

In addition to these problems, it would be a good idea to look over the “Check your understanding” problems from Chapters 5 through 10 AND the past quizzes and exams.

Solutions to the past quizzes and exams will be posted on our course website by the end of the last week of classes.

There will be some True/False questions on the exam, but otherwise all the problems will be “work out” questions (no multiple choice).

If you work some problem on your calculator, then **write down** some of your intermediate steps!!! Partial credit will be awarded for wrong answers, but only if you show your work. NOTE: credit WILL NOT be rewarded for correct answers showing no work.)

**Submit** solutions to these problems on **Thursday (12/08)** for your final quiz grade.
1. Let \( N(t) \) denote the size of a population at time \( t \) and assume that

\[
\frac{dN}{dt} = f(t)
\]

for some function \( f \).

(a) Express the cumulative change in the population size in the interval \([0, 3]\).

(b) If \( f(t) = 3 - t^2 \), which number do you expect to be larger, \( N(1.9) \) or \( N(2.1) \)? Justify your answer. (Hint: is \( f \) positive at \( t = 2 \)?)

2. Gaastra (1959) measured the effects of atmospheric \( \text{CO}_2 \) enrichment on \( \text{CO}_2 \) fixation in sugar beet leaves at various light levels. He found that increasing \( \text{CO}_2 \) at fixed light levels increases the fixation rate and that increasing the light levels at fixed atmospheric \( \text{CO}_2 \) concentration also increased fixation. If \( F(A, I) \) denotes the fixation rate as a function of atmospheric \( \text{CO}_2 \) concentration \( (A) \) and light intensity \( (I) \), determine the signs of \( \partial F/\partial A \) and \( \partial F/\partial I \).

3. Suppose that the relative growth rate of a plant is 10%; that is, if \( B(t) \) denotes the biomass at time \( t \), then

\[
\frac{1}{B(t)} \frac{dB}{dt} = 0.1.
\]

Suppose that the biomass at time \( t = 1 \) is equal to 5 grams. Use a linear approximation to compute the biomass at time \( t = 1.1 \).

4. Compute the integrals exactly (by hand!), showing all steps.

(a) \( \int \sin(2x)dx \)

(b) \( \int x \sin(2x)dx \)

(c) \( \int_3^4 (2x + 4) \cos(x^2 + 4x)(\sin(x^2 + 4x))^{10} dx \)

5. The survival function \( S(x) \) is the probability that the individual is alive at age \( x \) (in years).

(a) Explain why \( S(x) = 1 - F(x) \), where \( F \) is the cumulative distribution function for mortality.

(b) The hazard rate \( \theta(x) \) is defined as the relative rate of decline of the survival function:

\[
\theta(x) = -\frac{1}{S(x)} \frac{dS}{dx}.
\]

Explain why \( \theta \) is always nonnegative.

(c) If \( \theta(x) = 2 \), what is the probability that the individual will live more than ten years?

6. In a study of \( \text{Drosophila melanogaster} \) by Mackey (1984), the number of bristles on the fifth abdominal sternite in males was shown to follow a normal distribution with mean 18.7 and standard deviation 2.1. Find an interval centered at the mean so that approximately 95% of the population have bristle numbers that fall into this interval.
7. The four systems below describe the relationship between two populations.

(a) For each system, match the system of differential equations with the most likely slope field.  
(Hint: $P$ is always the horizontal axis.)

I. \[
\begin{align*}
\frac{dP}{dt} &= 1.4P - 0.8PK \\
\frac{dK}{dt} &= -1.6K + 0.7PK
\end{align*}
\]

II. \[
\begin{align*}
\frac{dP}{dt} &= 0.4PK - P \\
\frac{dK}{dt} &= -0.4PK
\end{align*}
\]

III. \[
\begin{align*}
\frac{dP}{dt} &= 0.2P - 0.5PK \\
\frac{dK}{dt} &= 0.6K - 0.8PK
\end{align*}
\]

CONNECT THE DOTS

A.  
B.  
C.  
D.  

(b) • Which looks most like a predator-prey model? (Identify which is the prey.)
• Which looks most like a spread-of-disease model? (Identify the susceptible population.)
• Which looks most like a model of two populations competing for the same resources?

8. Hydrocodone bitartrate is used as a cough suppressant. After the drug is fully absorbed, the quantity of drug in the body decreases at a rate proportional to the amount left in the body, with constant of proportionality $k$. Suppose that the half-life of hydrocodone bitartrate in the body is 3.8 hours, and that the oral dose taken in 9 mg.

(a) Write a differential equation for the quantity $Q$ of hydrocodone bitartrate in the body at time $t$, in hours, since the drug was fully absorbed.

(b) Solve your differential equation, assuming that at $t = 0$ the patient has just absorbed the full 9 mg dose of the drug. Your answer should not involve any arbitrary constants.

(c) How much of the 9 mg dose is still in the body after 12 hours? (include units)
9. Consider the function

\[ f(x, y) = \frac{6 - y}{x} . \]

Draw and label three level-curves for the function \( f \) on the graph below.

10. The probability density function for the time it takes a flower to see its 4th pollinator is given by

\[ p(t) = \frac{2^4 t^3}{6} e^{-2t} \quad (t \geq 0) . \]

(a) Write-out/describe, explicitly, the two calculations that must be done to compute the median and mean time to pollination.

(b) Determine the mean and median for this function.

11. Find all equilibrium solutions for the equation

\[ \frac{dy}{dx} = 0.5y(y - 4)(2 + y) . \]

12. A microbiologist must prepare a culture medium in which to grow a certain type of bacteria. The percent of salt contained in this medium is given by

\[ S = 6xy^2 , \]

where \( x \) and \( y \) are the nutrient solutions to be mixed in the medium. For the bacteria to grow, the medium must be 18% salt. Nutrient solutions \( x \) and \( y \) cost $1 and $3 respectively. How much of each nutrient solution should be used to minimize the cost of the culture medium? What is the practical meaning of \( \lambda = 0.0467 \)?
13. True or False?

( T / F )  Given \( \begin{array}{c|cc}
0 & 1 & 2 \\
1 & 0 & 1 \\
3 & 0 & 1 \\
\end{array} \) for a function \( f \), 5 is a Riemann sum approximation for \( \int_{0}^{3} f(x)dx \).

( T / F )  Let \( v(t) \) denote the velocity (in mph) of your car on a drive from Chicago to Duluth, MN. If \( \frac{1}{7} \int_{0}^{7} v(t)dt = 65 \), then you broke the law at some point.

( T / F )  You deposit money into your account at a constant rate of $200 per year. If the bank account accrues interest continuously at an annual rate of 3%, then you will have more than $402 after two years.

( T / F )  The substitution \( w = (q + 3)^{10} \) converts \( \int (q + 3)^{10} dq \) into \( \int w dw \).

( T / F )  Suppose \( g(x, y) = 2x - y \) represents the growth rate of a plant, given \( x \) units of sunlight and \( y \) units of rain. This rate is often expressed as a function of \( \tilde{x} \) and \( \tilde{y} \), where: \( \tilde{x} \) is the difference between \( x \) and the average daily units of sunlight 5, and \( \tilde{y} \) is the difference between \( y \) and the average daily units of rain 0.1. An expression for \( g \) in terms of \( \tilde{x} \), \( \tilde{y} \) is \( 2\tilde{x} - 10 - \tilde{y} - 0.1 \).

( T / F )  If \((x_0, y_0)\) is a critical point of \( f \) and if \( f_{xx}, f_{xy} \) and \( f_{yy} \) are all zero at \((x_0, y_0)\), then \((x_0, y_0)\) is always a local maximum or a local minimum of \( f \).

( T / F )  If \( f(x, y) = 3x^2e^{2y} \), then \( f_y(1, 0) = f_x(1, 0) \).

( T / F )  Circles centered at the origin are solutions to the differential equation \( dy/dx = -x/y \).

( T / F )  If \( dy/dx = -xy \) and \( y(2) = -1 \), then \( y(2.1) \approx -0.8 \).

( T / F )  The slope field of the differential equation \( dy/dx = -x/(1 + y) \) has slope 3 at the point \((6, 1)\).

( T / F )  \( \int_{0}^{1} x^2 dx > \int_{0}^{1} \sqrt{x} dx \).

( T / F )  In a dual population model, assume

\[
\begin{align*}
dx/dt &= -3x + xy \\
dy/dt &= 2y - 5xy
\end{align*}
\]

If the initial populations are \( x = 1 \) and \( y = 2 \), then both populations will decrease over time.