Quiz #9

Applied Calculus II – Math 132.003 – Fall 2011

Date: 11/10/11  Names: ________________________________

Show your work for credit.

1. (5 pts) A microbiologist must prepare a culture medium in which to grow a certain type of bacteria. The percent of salt contained in this medium is given by

\[ S = 6xy, \]

where \( x \) and \( y \) are the nutrient solutions to be mixed in the medium. For the bacteria to grow, the medium must be 12% salt. Nutrient solutions \( x \) and \( y \) costs $1 and $2, respectively. How much of each nutrient solution should be used to minimize the cost of the culture medium?

\[
\begin{align*}
\mathcal{f}(x,y) &= x + 2y \\
g(x,y) &= 6xy = 12 \\
\mathcal{f}_x &= \lambda (6y) \\
\mathcal{f}_y &= 2 = \lambda (6x) \\
6xy &= 12
\end{align*}
\]

\[
\begin{align*}
\lambda &= \frac{1}{6y} \\
2 &= (\frac{1}{6y}) 6x \\
x &= 2y \\
6 (2y)y &= 12 \\
y^2 &= 1 \\
y &= \pm 1 \\
&\Rightarrow y = 1.
\end{align*}
\]

Then,\( \mathcal{f}_x \Rightarrow x = 2 \cdot \quad CP = (2, 1)\)
2. (5 pts) You have 10 hours to devote to studying for two assignments, worth 20 points and 50 points, respectively. You figure that an hour devoted to the second assignment is half as effective as an hour devoted to the first assignment. That is, if $P(h_1, h_2)$ is the total number of points you can receive, then

$$P(h_1, h_2) = 20(1 - e^{-h_1}) + 50(1 - e^{-0.5h_2}).$$

(a) What is the objective function and what is the constraint equation? (These are two different questions!)

Objective function: points, $P(h_1, h_2) = 20(1 - e^{-h_1}) + 50(1 - e^{-0.5h_2})$

Constraint equation: hours, $h_1 + h_2 = 10$

(b) Use the method of Lagrange multipliers to show that $(h_1, h_2, \lambda) = (3.18, 6.82, 0.83)$ corresponds to the constrained maximum.

1. $20 e^{-h_1} = \lambda (1)$
   \[\Rightarrow 20 e^{-h_1} = 25 e^{-0.5h_2} \Rightarrow e^{-0.5h_2 - h_1} = \frac{25}{20}\]
   \[\Rightarrow \frac{1}{2} h_2 - h_1 = \ln \left(\frac{5}{4}\right)\]

2. $25 e^{-0.5h_2} = \lambda (1)$
   \[\Rightarrow h_2 = 2h_1 + 2 \cdot 2.223\]

3. $h_1 + h_2 = 10$

   \[\Rightarrow h_1 + (2h_1 + 0.446) = 10 \Rightarrow h_1 = 3.18. \text{ Then } h_2 = 6.81\]

(c) In practical terms, what does $\lambda = 0.83$ mean?

Then $1 \Rightarrow \lambda = 0.83$

Studying for an extra hour would produce (roughly)

$$\Delta P \approx \lambda \cdot \Delta H = 0.83 \cdot 1 = 0.83 \text{ more points.}$$