Advice/Comments. Practice, practice, practice. You need to be able to solve analytically, quickly and effortlessly, many types of definite and indefinite integrals (i.e., using shortcuts and methods such as integration by parts and substitution). The book has plenty of examples—in the exercises at the end of each section and at the end of each chapter—so I won’t bother to give any below. It is essential that you become and integrating machine by Thursday, February 24, because the majority of the exam will go beyond this, testing your ability to approximate, apply, and interpret the integrals (definite and indefinite) in ways that will require much deeper critical thinking and problem solving. (Failure to have mastered the rote calculations will cause you to waste much-needed time for deliberation during the exam!)

1. Check your understanding with the “Check Your Understanding” problems at the end of each chapter.
2. Synthesis/Concept questions are found throughout the “Review Problems” at the end of each chapter.
3. Standard integration problems in the text:
   - Section 7.1: 1–68 are all standard problems.
   - Section 7.2: 1–43 standard practice with substitution
   - Section 7.3: 1–25 standard practice with definite integrals
   - Section 7.3: 26–35 practice with calculating exact areas using definite integrals
   - Section 7.4: 1–21 standard practice with integration by parts
   - Chapter 7 Review (pp. 322–323): 1–41 Even more practice!

Keep in mind that on the exam you will not be told which method to use. So as you do these problems you should think to yourself why a certain method applies or does not apply to a particular integral. (This is not an easy task!)

Below, I list some practice problems from Chapters 6 & 7 of the type that I alluded to above in my Advice/Comments. (These were created by Adam Spiegler.) Please have them worked by Tuesday, if possible, so we can discuss any questions you have during class.

4. Which of the following is equivalent to $f(x) = \ln(x^5 + 1) + C$?

   (a) $\int \frac{5x^4}{x^5 + x} \, dx$
   (b) $\int \frac{5x^4}{x^5 + 1} \, dx$
   (c) $\int \frac{\ln(x^5 + 1)}{x^5 + 1} \, dx$
   (d) $\int \frac{\ln(x^5 + 1)}{x^6 + x} \, dx$
5. Decide what method of integration (substitution, integration by parts, or nothing fancy) are appropriate for evaluating each of the indefinite integrals below (feel free to go on and evaluate each indefinite integral if you like!).

\[ \int x^2 e^{2x^3} \, dx \quad (a) \quad \int \frac{t^4 + 1}{t^3} \, dt \quad (c) \quad \int 2 \sin(\theta^2) \, d\theta \]

\[ \int \frac{t^3}{t^4 + 1} \, dt \quad (b) \quad \int 2\theta^2 \sin(\theta) \, d\theta \quad (d) \quad \int \sin(2\theta) \, d\theta \quad (f) \]

6. Fuel pressure in the fuel tank of a space shuttle is decreasing at a rate of \( r(t) = 13e^{-0.1t} \) psi per second at time \( t \) seconds after lift off.

(a) At what rate is the pressure decreasing 20 seconds after lift off?
(b) What is the total change in pressure in the fuel tank over the first 20 seconds after lift off?

7. Find the exact value of the constant \( b > 0 \) such that the area under the graph of \( f(x) = 3x^2 \) between \( x = 0 \) and \( x = b \) is 8.

8. Decide whether the improper integral \( \int_{-\infty}^{1} e^{2x} \, dx \) converges or diverges. If it converges, find the exact value the that it converges to.

9. At time \( t \) hours after taking a tablet, the rate at which a drug is being eliminated is

\[ r(t) = 50 \left( e^{-0.1t} - e^{-0.2t} \right) \]

Assuming that eventually all of the drug is eliminated, calculate the original dose.

10. Below is the graph of a derivative, \( f'(x) \).

![Graph of the derivative of f(x)](image)

(a) Estimate \( \frac{f'(3) - f'(0)}{3 - 0} \).

(b) Estimate \( \frac{f(3) - f(0)}{3 - 0} \).

11. From the time a child is born until he is 18, a father plans to set aside \$100 times the child’s current age each year.

(a) Find a formula for the income stream \( S(t) \) in dollars \( t \) years since the child is born.
(b) How much money will be in the account when the child turns 18 years old?
12. The figure below shows the graph of a derivative \( f'(x) \).

![Figure 2: Graph of the DERIVATIVE of \( f(x) \)](image)

(a) Where does \( f(x) \) have a local maxima? local minima?
(b) Over what interval(s) is \( f(x) \) concave up? concave down’?

13. Given the values of \( f'(x) \) in the table and that \( f(0) = 40 \), find the best possible estimate for \( f(6) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) )</td>
<td>3</td>
<td>15</td>
<td>27</td>
<td>29</td>
</tr>
</tbody>
</table>

14. Approximate the average of \( h(x) \) over the interval \( 100 \leq x \leq 350 \).

![Figure 2](image)

15. A lottery winner is offered a choice between
   (a) a lump sum of $50,000 now, or
   (b) $5000 per year for 15 years.
   
   If the interest rate is 5%, compounded continuously, which is a better choice? Answer A or B.
16. Find the average value of \( f(x) \) between \( x = 0 \) and \( x = 20 \), and use your approximate average value to approximate \( \int_0^{20} f(x) \, dx \).

\[ f(x) \]

17. The initial population of a bacteria colony is 200 bacteria and growing at a continuous rate of 24% per hour. Let \( P \) denote the size of the bacteria colony \( t \) hours since the colony was formed.

(a) Find a formula to express \( P \) as a function of \( t \).
(b) What is the average rate of change of the colony over the first 24 hours?
(c) What is the average size of the colony over the first 24 hours?

18. The following table shows the market value, in thousands of dollars, of a single family home.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>value (in thousands of $)</td>
<td>141</td>
<td>146</td>
<td>153</td>
<td>169</td>
<td>235</td>
</tr>
</tbody>
</table>

Find (or approximate) the following:

(a) The absolute change in value of the home over the period from 2000 to 2008.
(b) The absolute rate of change of the value of the home in 2008.
(c) The percent change in the value of the home over the period from 2000 to 2008.
(d) The relative rate of change of the home’s value in 2008.

19. The dropout rate at a high school has been declining since 2002. If \( P(t) \) is the number of high school dropouts as a function of the year \( t \). The table below gives values of the relative rate of change, \( P'(t)/P(t) \) each year.

<table>
<thead>
<tr>
<th>Year</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P'(t)/P(t) )</td>
<td>-0.021</td>
<td>-0.024</td>
<td>-0.037</td>
<td>-0.040</td>
<td>-0.032</td>
<td>-0.035</td>
</tr>
</tbody>
</table>

Approximate the total percent change in the dropout rate over the years 2002 to 2007.