Show your work for credit.

1. (5 pts) Pollution is being dumped into a lake at a rate \( r(t) \) which is increasing at a constant rate from 10kg/year to 50kg/year until a total of 270kg has been dumped.

   (a) On the axes below, sketch a graph of \( r(t) \).

   ![Graph of r(t)](image)

   We don’t know when the dumping reaches 270 kg, only that the dump-rate reaches 50 at that point. Since the dumping-rate \( r(t) \) is increasing at a constant rate, we have drawn a straight line.

   (b) Find the formula for \( r(t) \).

   The slope of the line is \( m = \frac{50 - 10}{T - 0} \). So the equation is \( r(t) = 10 + \frac{40}{T} t \).

   We can solve for \( T \) using the formula for the area of a trapezoid: \( 270 = \frac{1}{2} (10 + 50) T \), or \( T = 9 \). Thus, \( r(t) = 10 + \frac{40}{9} t \)

   (c) How long does it take until 270kg of pollution has been dumped? (Assume the lake has no pollution before the dumping begins.)

   They are just looking for what we call \( T \), so the answer is 9.
2. (5 pts) The graph of \( f(x) = 3x^2 - 3c^2 \) is shown in the figure. Find the total area enclosed between \( f(x) \), the \( x \)-axis, \( x = 0 \) and \( x = 3 \). Assume \( c \) is a positive constant.

In order to make both regions count as positive area, we can’t just integrate from 0 to 3. We need to solve for \( X \) and then compute:

\[
\text{total area} = \int_0^X (-f(x)) \, dx + \int_X^3 f(x) \, dx.
\]

Putting \( 3x^2 - 3c^2 = 0 \), we find that \( x = \pm c \), so \( X = c \).

Next, find an antiderivative of \( f(x) \). We get \( F(x) = x^3 - 3c^2x \), so the answer is:

\[
\text{total area} = \left\{ -(c^3 - 3c^3) + (0^3 - 3c^20) \right\} + \left\{ (3^3 - 3c^23) - (c^3 - 3c^3) \right\} = 27 + 4c^3 - 9c^2.
\]