1. (5 pts) There are about 4480 manatees in the United States today. New calves are born about every two to five years. Assuming half the population is female and all are of calfbearing age, we may claim a very optimistic renewal rate of \( r(t) = 1120t \). If about 13\% of the population fails to survive each year (due to old age, boating accidents, oil pollution, etc.), what will be the population in ten years?

This is a future value problem. Survival rate of the population is 87\%. Renewal rate is 1120\( t \). (Note: this is obnoxiously optimistic! A rate of 1120 would be more reasonable.) Thus we have the scenario:

\[
FV = 4480(0.87)^{10} + (0.87)^{10} \int_0^{10} (1120t)(0.87)^{-t} \, dt.
\]

Thus \( FV \approx 4480(0.2484) + (0.2484)(149021) = 38130 \).
2. (5 pts) The time between arrivals of insect pollinators to a flowering plant is generally assumed to be exponentially distributed. This means that the density function is given by
\[ p(t) = p_1(t) = \lambda e^{-\lambda t} \quad (t \geq 0) \]
for some parameter \( \lambda \). (We’ll take \( \lambda = 0.5/hr. \)) Under this assumption, it can be shown that the waiting time for the \( n \)th arrival of a pollinator is
\[ p_n(t) = \frac{\lambda^n t^{n-1}}{(n-1)!} e^{-\lambda t} \quad (t \geq 0). \]

(a) Sketch the first few graphs \( p_n(t) \) on the same axes.
Here are \( p_1(t) \), \( p_2(t) \), and \( p_3(t) \):

(b) Use the graphs to estimate the mean and median waiting time for the corresponding pollinators. Point out any trend(s) you see (among the medians, among the means, or between the medians and means). Use your calculator to check your impression(s) by computing the mean and median for the 5th, 15th, and 50th pollinator.

The point of this problem was: (i) to see that the mean was always to the right of the median (because of the long tail of outliers); (ii) to see that eventually the median and the mode are essentially the same...because the \( n \)th pollinator graph eventually looks like a normal distribution. (Remember, we said that “everything is normal.” Here is another example.)

I won’t bother to estimate the medians/means, I’ll just graph \( p_9(t), p_{20}(t) \), and \( p_{30}(t) \) on the same graph to show how they are looking more and more like normal distributions:
(c) Now compute the mean and median, explicitly and precisely, for $p_1(t)$.

**Median:** Solve for $T$ in $\frac{1}{2} = \int_{0}^{T} 0.5 e^{-0.5t} \, dt$.

$$\frac{1}{2} = -e^{-0.5T}\bigg|_{0}^{T} = -e^{-0.5T} + e^{0}$$

$$\frac{1}{2} = -e^{-0.5T}$$

$$\frac{\ln(1/2)}{-0.5} = T$$

$$T \approx 1.386.$$

**Mean:** Compute $I = \int_{0}^{\infty} t(0.5 e^{-0.5t}) \, dt$. We’ll use integration by parts, taking $u = t$ and $dv = 0.5 e^{-0.5t} \, dt$. Thus $du = dt$ and $v = -e^{-0.5t}$.

$$I = uv\bigg|_{0}^{\infty} - \int_{0}^{\infty} vdu$$

$$= -te^{-0.5t}\bigg|_{0}^{\infty} + (-2)e^{-0.5t}\bigg|_{0}^{\infty}$$

$$= -e^{0.5t}(2 + t)\bigg|_{0}^{\infty}$$

$$= (-0) - (-1 \cdot 2)$$

because $e^{-0.5t}$ dies faster than $2 + t$ grows as $t \to \infty$, so

$$I = 2.$$