Write up your solution to ONE of these problems (announced in class and on the course web page) and submit it on Tuesday. Your solution should follow the Groupwork Write-up Guidelines & Hints (available on the course web page) as well as the ground rules laid out in the syllabus.

1 (Follow the line). Consider the curve defined by the equation $-8x^2 + 5xy + y^3 = -149$.
(a) Compute $dy/dx$.
(b) Write an equation for the tangent line to the curve at the point $(4, -1)$.
(c) There is a number $k$ such that the point $(4, 2, k)$ is on the curve. Using the tangent line found in (b), approximate the value of $k$.

2 (Where ya headed?). An unidentified object moves along the $s$-axis, with displacement $s = s(t)$ (meters), velocity $v = v(t)$ (m/sec) and acceleration $a = a(t)$ (m/sec$^2$). It so happens that the velocity and displacement are related by the equation $v = \sqrt{8s + 16}$. Also, $s(0) = 6$.
(a) Show that $a$ is constant, and find its value.
(b) Graph $v$ as a function of $s$.
(c) Graph $v$ as a function of $t$.

3 (When will you get there?). An object is moving along the parabola $y = 3x^2$.
(a) When it passes through the point $(2, 12)$, its “horizontal” velocity is $dx/dt = 3$. What is its “vertical” velocity at that instant?
(b) If it travels in such a way that $dx/dt = 3$ for all $t$, then what happens to $dy/dt$ as $t \to +\infty$?
(c) If, however, it travels in such a way that $dy/dt$ remains constant, then what happens to $dx/dt$ as $t \to +\infty$?

4 (Shoot now!). A soccer player is dribbling while running parallel to the sideline and directly toward the point on the goal line which is 30′ from the near edge of the goal. The goal mouth is 20′ wide. Does the goal mouth seem to be getting bigger or smaller to her (that is, is the angle $\theta$ increasing or decreasing) when she is (a) 40′ from the goal line; (b) 35′ from the goal line; (c) 30′ from the goal line? Make sure that your answers fit with common sense. Hint: What is $d\theta/dt$?

5 (It’s not strange, it’s natural). The following statements are true:

$(10, 000, 000, 000)\left(\frac{1}{10, 000, 000}\right) = 1.000000025850956\cdots$ and $\ln 10 = 2.302585092\cdots$

Explain the amazing coincidence of the digits. Hint: Use the linearization of $e^x$ around $x = 0$.

6 (Blow it Up). A spherical balloon is being inflated with hot air. At time $t = 5$ seconds the radius of the balloon is equal to 10cm and is increasing at the instantaneous rate of 3 cm/sec. (a) How fast is the volume changing? (b) Use differentials to find an approximate value of the volume of the balloon at time $t = 5.1$ seconds.