Groupwork Write-up Guidelines & Hints

Your write-ups will be graded on a scale of 0-10. The grade will indicate the correctness of the mathematical content and the manner in which the math is presented in your write-up. Since you may never have been graded on how you present your ideas in math, here are some guidelines.

Look at one of the examples in the book. The authors begin by writing a statement of the problem. They use complete English sentences. They explain those steps which are not obvious to a calculus student (and don’t explain the steps that are). If there’s a graph, they label it, and they discuss what can be deduced from the graph in the context of the example. At the end, they state the conclusion.

Don’t be discouraged if your initial write-ups receive low grades because of poor exposition. Writing solutions with explanations is probably not something you have done in a math class before. You will improve. Because mathematics is used to solve problems and explain the solutions to others, writing clear solutions is a good habit to develop. Creating a good write-up forces you to think more carefully about how you did the problem—and therefore helps you learn calculus.

Below are some of the mistakes I consistently see when grading workshops. This means YOU!

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Precision, part I Here is a joke:

A mathematician, a physicist, and an engineer are visiting Scotland for the first time and riding through the countryside by train. “Oh!” the engineer says upon seeing a sheep on a hill, “the sheep in Scotland are black!” The physicists chimes in, “no, no, that sheep is black.” “Well,” the mathematician adds, “that side of that sheep is black.”

It may not be very funny to you, but it is because you don’t know many mathematicians yet. Like physicists and engineers, we develop tools and ideas to solve problems. Unlike these individuals, we like to know things for certain and only claim things we certainly know. Precision is our bread-and-butter. We take care to be very precise—both with our language and with our reasoning. This is reflected in our grammar in the following way:

$$\pi = 3.14159 \text{ FALSE!}$$

$$\pi \approx 3.14159 \text{ TRUE.}$$

Don’t say things that aren’t true when solving a mathematical problem.

Sentences (Symbols, part I) When reducing a mathematical expression, use the “=” sign only when two things are equal, and the “⇒” sign only when the second statement is a direct consequence of the first:

<table>
<thead>
<tr>
<th>Good Uses</th>
<th>Bad Uses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^2 + n^2 = 2n^2$</td>
<td>$n^2 + n^2 = 2n^2$</td>
</tr>
<tr>
<td>$7x = 3 \implies x = \frac{3}{7}$</td>
<td>$7x = 3 = x = \frac{3}{7}$</td>
</tr>
</tbody>
</table>

Here are two english-language sentence analogs of the bad uses:

- Apples and apples, two apples. Alexis is the name of the cat is named sophie.

Practice good mathematical grammar; know what the elements of your sentence mean.

Symbols, part II Mathematicians invented algebra so we wouldn’t have to write sentences like this:

If thou art diligent and wise, O stranger, compute the number of cattle of the Sun, who once upon a time grazed on the fields of the Thrinacian isle of Sicily, divided into four herds of different colours, one milk white, another a glossy black, a third yellow and the last dappled. In each herd were bulls, mighty in number according to these proportions: Understand, stranger, that the white bulls were equal to a half and a third of the black together with the whole of the yellow, while the black were equal to the fourth part of the dappled and a fifth, together with, once more, the whole of the yellow. … But come,
understand also all these conditions regarding the cattle of the Sun. When the white bulls mingled their number with the black, they stood firm, equal in depth and breadth, and the plains of Thrinacia, stretching far in all ways, were filled with their multitude. . . . —Archimedes, 200 B.C.

Full text available http://www.mcs.drexel.edu/~crorres/Archimedes/Cattle/Statement.html

Instead we write something like,

Let \( W, B, Y, D \) be the number of cattle coloured white, black, yellow, and dappled, respectively. Let \( w, b, y, d \) be the number of bull in each herd. The numbers satisfy:

\[
\begin{align*}
w & = \frac{5b}{9} + y \\
b & = \frac{9}{20}d + y \\
& \vdots \\
w + b & = N^2 \\
& \vdots
\end{align*}
\]

where \( N \) is some positive integer.

If you have a quantity you’re considering, don’t be afraid to give it a variable name if it will help with the clarity of your exposition. With that said, PLEASE don’t leave variables undefined, it is sloppy and confusing (though rarely as confusing as the passage above).

**Consistency** Regarding symbol choices, if you name the hypotenuse of a triangle \( C \) at some point, do not later on refer to it as \( c \) (or \( x! \)).

Give every quantity one name (symbol) and one name only.

**Clarity, part I** Note the use of \( Y \) and \( y \) in the cattle problem above. These are related quantities so they are given related names. Also, they are codes in the sense that, as best as possible, the symbols chosen reflect something essential about what they represent ("yellow" things). Similarly, a formula for the volume of a box in terms of its height should begin “\( V(h) = . . . \)”, not “\( f(x) = . . . \)”. Reserve similar symbols for related quantities and choose your symbols well.

**Catching Your Breath** The language of mathematics packs a lot of information into a small space. This makes reading a half-page of equations fairly difficult. The conscientious writer recognizes this.

Pause from time to time and write a sentence telling the reader where he is heading next.

**Clarity, part II** How would you feel listening to the news if every time someone mentioned Lady Gaga, they suffixed it with the phrase “the outrageous and multi-talented chanteuse from NYC?” Pretty insulting right? We all know this, get to the point! Similarly, don’t write

\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{h}{y}.
\]

Omit the middle step, we all know this and saying it just detracts from the readability of your presentation. And by all means, avoid big scratch symbols. Such clutter rarely adds to the clarity of an exposition. For example, instead of writing

\[
\frac{32x^2(x + 2)^2}{16x^9(x + 2)^2} = \frac{6(x + 2)(x - 2)}{3(x - 2)},
\]

simply write

\[
\frac{32x^7(x + 2)^3}{16x^9(x + 2)^2} = \frac{6(x + 2)(x - 2)}{x^2}.
\]

**Clutter does not engender clarity.**