Quiz #2 - Solutions
Calculus I – Math 161.009 – Fall 2010

Name: ____________________________

1. (6 pts) If \( f(a) = 3 \) and \( f'(a) = 5 \), determine the equation of the tangent line passing through the curve \( h(x) = 3f(x) + 2 \) at \( x = a \). (Hint: your answer will have the parameter \( a \) in it.)

The value of \( h(x) \) at \( x = a \) is \( h(a) = 3f(a) + 2 = 3 \cdot (3) + 2 = 11 \). The slope of \( h(x) \) at \( x = a \) is computed as follows:

\[
h'(a) = h'(x)|_{x=a} = \frac{d}{dx}(3f(x) + 2)|_{x=a} = (3f'(x) + 0)|_{x=a} = 3f'(a) = 3 \cdot 5 = 15.
\]

Using point-slope form for lines, conclude that tangent line to \( h \) at \( x = a \) is given by

\[
y - 11 = 15(x - a).
\]

2. (4 pts) Calculate these four limits using theorems from class (not the formal \( \epsilon-\delta \) definition).

(a) \[ \lim_{x \to \infty} \frac{x - 7}{3x^2 + 4} \]

\[
\frac{x - 7}{3x^2 + 4} = \frac{1/x - 7/x^2}{3 + 4/x^2}, \text{ so } \lim_{x \to \infty} \left( \ldots \right) = \frac{\lim_{x \to \infty} (1/x) - \lim_{x \to \infty} (7/x^2)}{3 - \lim_{x \to \infty} 4/x^2} = \frac{0 - 0}{3} = 0.
\]

(b) \[ \lim_{x \to 0^+} \frac{(x - 1) \sin x}{x^2} \]

\[
\frac{(x - 1) \sin x}{x^2} = \frac{x - 1}{x} \cdot \frac{\sin x}{x}, \text{ so } \lim_{x \to 0^+} \left( \ldots \right) = \left( \lim_{x \to 0^+} \frac{x - 1}{x} \right) \left( \lim_{x \to 0^+} \frac{\sin x}{x} \right) = (-\infty)(1) = -\infty.
\]

(c) \[ \lim_{x \to 2^-} \frac{x - 7}{x^2 - 4} \]

\[
\frac{x - 7}{x^2 - 4} = \frac{x - 7}{x + 2} \cdot \frac{1}{x - 2}, \text{ so } \lim_{x \to 2^-} \left( \ldots \right) = \left( \lim_{x \to 2^-} \frac{x - 7}{x + 2} \right) \left( \lim_{x \to 2^-} \frac{1}{x - 2} \right) = (-3/4)(-\infty) = \infty.
\]

Second factor is about as simple as it gets—no need to use one of my simplification tricks—but you may look at limits of numerator and denominator separately if you like:

\[
\lim_{x \to 2^-} \frac{1}{x - 2} = \frac{\lim_{x \to 2^-} 1}{\lim_{x \to 2^-} (x - 2)} = \frac{1}{0'},
\]

where the denominator \( \to 0' \) through negative numbers. Hence, the answer is \(-\infty\).

(d) \[ \lim_{x \to 2^-} \frac{x - 7}{x^2 - 4} \]

Use same analysis as in Part (c) to compute the right-hand limit. Deduce that the two-sided limit is undefined (or "Does Not Exist"): \[ \lim_{x \to 2^-} (\ldots) = -\infty, \text{ while } \lim_{x \to 2^+} (\ldots) = +\infty. \]