Chapter 1: Functions

1.
   (a) half-life = \( \frac{\ln(0.5)}{-0.000121} \approx 5728.5 \) years
   (b) It is about 12,146 years old

2.
   (a) \( g(t) = 10(\sqrt{1.2})^t \). So the population in 1990 was 10 million and it is growing at a rate of about 9.5% per year.
   (b) \( f(t) = 18 + \frac{3}{8}t \). The population was 18 million in 1990 and it is growing by 375,000 people per year.
   (c) The town which is growing exponentially will reach 25 million in about 10 years, in 2000. The town’s whose population is linear will reach 25 million in about 18.67 years. So sometime in 2008 but before 2009.

3. \( f(x) = -\frac{5}{9}x(x + 2)^2(x - 3) \)

4.
   (a) 17 meters
   (b) 10 meters
   (c) 14 meters
   (d) 13 hours

(e)
5. 

(a) There are several possibilities. Below are two possible transformations:
   i. Horizontal compression by a factor of $1/2$
   ii. Vertical reflection about the $x$-axis
   iii. Vertical shift up 3 units

Or also
   i. Vertical reflection about the $x$-axis
   ii. Vertical shift up 3 units
   iii. Horizontal compression by a factor of $1/2$

(b) Following the second set of transformations list above:

\[
(2, 25) \rightarrow (2, -25) \rightarrow (2, -22) \rightarrow (1, -22)
\]

(c) 

(d) \[g^{-1}(x) = \frac{\ln(-x + 3)}{2 \ln(5)} = \frac{\ln(-x + 3)}{\ln(25)} = \]

6. \[h(-x) = f(-x) - 2 = -f(x) - 2. \] He is neither even nor odd since \(h(-x) \neq h(x)\) and \(h(-x) \neq -h(x)\).

7. \[g(-x) = 2 - f(-x) = 2 - f(x) = g(x). \] Since \(g(-x) = g(x)\), \(g(x)\) must be even as well.
Chapter 2: Limits and Continuity

8. See odd answers in back of book.


10. \( \delta = \frac{4}{7} \)

11. \( \delta = 0.55 \)

12. \( \delta = \frac{6\epsilon - \epsilon^2}{4} \)

13. 4

14. \( -\frac{3}{2} \)

15. First note that \( f(x) \) is continuous over the interval \([-4, 4]\), so the IVT applies. Since \( f(-4) = -3 \) and \( f(-3) = 19 \), there must be some value of \( x \) in the interval \(-4 < x < -3\) such that \( f(x) = 0 \). Since \( f(0) = 1 \) and \( f(1) = -13 \), there must be a zero somewhere in the interval \( 0 < x < 1 \). Lastly, since \( f(3) = -17 \) and \( f(4) = 5 \), there must be another zero in the interval \( 3 < x < 4 \).

16. \( x = 3 \) and \( x = 1 \) are discontinuities. \( x = 3 \) is a removable discontinuity, and we could extend \( f(x) \) at \( x = 3 \) by setting \( f(3) = \frac{-18}{4} \). The discontinuity at \( x = 1 \) is not removable, it is a vertical asymptote.

17. .

(a) \( \lim_{x \to c} (3f(x) - 2g(x)) = 3 \lim_{x \to c} f(x) - \lim_{x \to c} g(x) = 3(12) - 2(-5) = 46. \)

(b) \( \lim_{x \to c} \left( \sqrt{f(x)} - g(x) \right) = \sqrt{\lim_{x \to c} f(x) - \lim_{x \to c} g(x)} = \sqrt{12 - (-5)} = \sqrt{17}. \)

18. \( \lim_{x \to -\infty} f(x) = -4. \)

19. \( \lim_{x \to -2^+} f(x) = -7. \) You cannot say anything for certain about the other one-sided limits.

20. Let \( h(x) = 2x \), which is continuous at \( x = 0 \). If \( f(x) = 1/x \), \( g(x) = 2x^2 \), then \( h(x) = f(x)g(x) = 2x \), however \( g(x) \) is not continuous at \( x = 0 \). This is not always true.

21. Not always true. Let \( h(x) = x^2 \) which is continuous at \( x = 0 \). If \( f(x) = 1/x \) and \( g(x) = x^2 - 1/x \), then \( f(x) + g(x) = x^2 = h(x) \) but both \( f(x) \) and \( g(x) \) are not continuous at \( x = 0 \).
22. True. We need to show \( \lim_{x \to c} h(x) = h(c) \) in order to prove \( h(x) \) is continuous at \( x = c \).

\[
\lim_{x \to c} h(x) = \lim_{x \to c} (f(x)g(x)) \\
= \left( \lim_{x \to c} f(x) \right) \cdot \left( \lim_{x \to c} g(x) \right) \\
= (f(c)) \cdot (g(c)) \quad \text{since } f(x) \text{ and } g(x) \text{ are continuous at } x = c \\
= h(c)
\]

Thus \( h(x) \) must be continuous at \( x = c \).

23. There are infinitely many different graphs you can draw. Below is one possible graph.

24. True. We are told \( f(x) \) is continuous on some interval we’ll call \( a \leq x \leq b \). If \( f(x) \) did change signs somewhere in this interval, there would be some value \( x_1 \) in this interval where \( f(x_1) > 0 \) and some value \( x_2 \) in this interval where \( f(x_2) < 0 \). By the IVT, there must therefore exist some value \( x = c \) between \( x_1 \) and \( x_2 \) such that \( f(c) = 0 \) which violates the assumption that \( f(x) \) is never zero in this interval.