Groupwork #2, Math 161 (Spring 2012)

Write up your solution to ONE of these problems (announced in class and on the course web page) and submit it on Wednesday. Your solution should follow the Groupwork Write-up Guidelines & Hints *(available on the course web page) as well as the ground rules laid out in the syllabus.*

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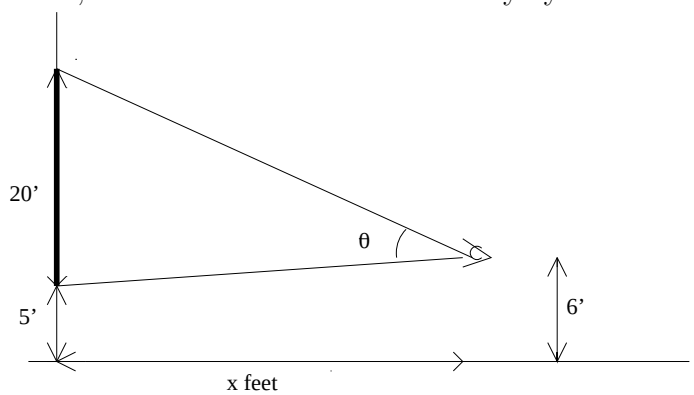
1 (Inverse Trig?). A large mural is $20'$ high and its bottom edge is $5'$ above floor level. My eye is $6'$ above the floor. Express my angle of view θ in terms of x, if I am standing x feet from the mural. (Hint: To get started, draw a horizontal line from my eye to the mural.)



2 (Connect the Dots). Let $f(x) = \begin{cases} 2x^2+2, & \text{if } x<1 \\ ax^2+bx, & \text{if } 1 \le x \le 2 \\ 10-\dfrac{2}{x}, & \text{if } x>2 \end{cases}$.

(a) Suppose that $a=2$ and $b=-3$. Graph $f(x)$ for $0 \le x \le 3$. Find the left and right hand limits of $f(x)$ as x approaches 1 and as x approaches 2.

(b) Find a and b so that the graph of $f(x)$ doesn't have any jumps.

3 (Unique. Smooth.). A continuation of Problem 2.

(a) For the a and b which you found in Part (b), find the slopes of the tangent lines to $2x^2+2$ and to ax^2+bx at $x=1$. What do your results tell you about the graph of f at $x=1$?

(b) Do a similar calculation for the graph at $x=2$.

4 (Definition of "Limit"). Let $f(x) = \dfrac{60}{x+3}$. Then $\lim_{x\to 2} f(x) = 12$. (*What limit laws allow you to make this conclusion?*) In this problem you will attempt to prove this limit statement using the ϵ-δ definition of a limit. Think of the graph of f in answering (a). Remember that the inequality $\left|\frac{60}{x+3} - 12\right| < \epsilon$ is equivalent to the inequality $12-\epsilon < \frac{60}{x+3} < 12+\epsilon$.

(a) Find numbers $a<2$ and $b>2$ such that $|f(x)-12| < 0.1$ for all x in the interval $a<x<b$.

(b) Using your answer to a), find a number $\delta_1 > 0$ such that $|f(x)-12| < 0.1$ whenever $|x-2| < \delta_1$. (*This is not complicated!*)

(c) Find a number $\delta_2 > 0$ such that $|f(x)-12| < 0.01$ whenever $|x-2| < \delta_2$. Repeat for two more (smaller and smaller) positive numbers ϵ of your own choosing, obtaining the corresponding δ_3 and δ_4.

(d) To conform to the definition of limit, do the following: Let any positive number ϵ be given. In terms of ϵ, find a number $\delta > 0$ such that $|f(x)-12| < \epsilon$ whenever $0 < |x-2| < \delta$.