# Groupwork #3, Math 161 (Spring 2012)

*Write up your solution to ONE of these problems (announced in class and on the course web page) and submit it on Wednesday. Your solution should follow the* Groupwork Write-up Guidelines & Hints *(available on the course web page) as well as the ground rules laid out in the syllabus.*

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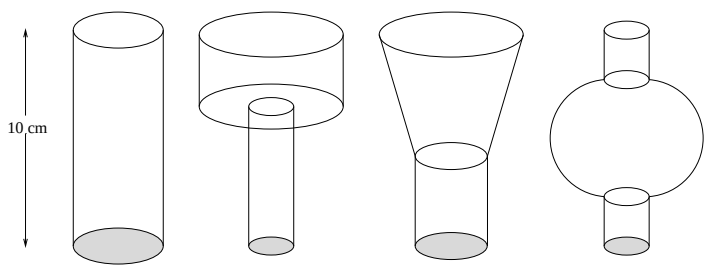
**1 (Cut it Up). (a)** Suppose that $F$ and $G$ are two continuous functions on an interval $a \le x \le b$, and that $F(a) \le G(a)$ but $F(b) \ge G(b)$. Show that the equation $F(x) = G(x)$ is satisfied for some $x$ on the interval. (Hint: apply the intermediate value theorem to a suitable combination of $F$ and $G$.) **(b)** By applying (a), show it is possible to cut any circular cake through its exact center so that the two halves have exactly the same amount (area) of icing, no matter how unevenly the cake may have been iced.

**2 (Fill 'er Up).** Four containers are each 10 cm tall, and each of them has a volume of 24 cm$^3$. They are each being filled by a liquid at the rate of 8 cm$^3$ per minute. Here is a picture of the four containers:




(A) (B) (C) (D)

**(a)** For each of the containers, graph the height, $h(t)$, of the level of the liquid in the containers measured in centimeters as a function of time, $t$, measured in minutes. **(b)** Which of the functions graphed in a) are continuous? Justify your conclusions as well as you can. **(c)** Which of the functions graphed in a) are differentiable? Justify your conclusions as well as you can.

*Special note: this liquid is ideal, so please assume there's no surface tension and no bubbles and no magic and ...*

**3 (Snuggle Up).** Find the constant $c$ so that the parabola $y = x^2 + c$ rests snugly in the $V$ formed above the $x$-axis by the two lines $y = \pm 3x$.

**4 (Clean Up (that spot on the ceiling)).** At time $t = 0$, Irwin fires a rubber band at a spider on the ceiling. At time $t = 3$ it hits the mark; then it falls. Let $s(t)$ represent the height of the rubber band above the floor at time $t$. **(a)** Which is larger, $s(1)$ or $s(2)$? **(b)** Which is larger, $s'(1)$ or $s'(2)$? **(c)** Sketch possible graphs of $s$ and $s'$ for $0 \le t \le 4$.

**5 (Stack 'em Up).** Using the figure at right, determine $\lim_{x \to a} \dfrac{f(x)}{g(x)}$:

(a) using a geometric argument;
(b) using limits, and the definition of derivative.

*Hint: you may prefer to use the formulation* $\lim_{x \to b} \frac{f(x)-f(b)}{x-b}$ *instead of the formulation* $\lim_{h \to 0} \frac{f(b+h)-f(b)}{h}$.