Groupwork #4, Math 161 (Spring 2012)

Write up your solution to ONE of these problems (announced in class and on the course web page) and submit it on Wednesday. Your solution should follow the Groupwork Write-up Guidelines & Hints *(available on the course web page) as well as the ground rules laid out in the syllabus.*

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1 (How High?). Consider $f(x) = \dfrac{5x^2 - 10x}{e^x}$, $x \geq 0$.

(a) Graph $y = f(x)$ in the window $0 \leq x \leq 3$ and $-3 \leq y \leq 1$. Locate the apparent highest and lowest points on the curve.

(b) Calculate $f'(x)$ and use it to locate (algebraically) all those values of x at which the graph has a horizontal tangent line. Check your answer against a).

(c) Use $f'(x)$ to find an equation for the line that is tangent to the curve $y = f(x)$ at $x = 1$. Draw the line on the graph in a) to check the result.

(d) Use the graph in a) to guess the values of x where $f'(x)$ is largest and where $f'(x)$ is smallest. Then graph the equation $y = f'(x)$ on your calculator to check your guesses.

2 (Watch Out!). Two particles P_1 and P_2 move along the s-axis starting at time $t = 0$. Their respective position functions are $s_1(t) = 8e^{2t} - e^{4t}$ and $s_2(t) = \sin^2 2t - \sin^2 t$.

(a) Find on your calculator a numerical approximation for the time and place at which they (first) collide. Is it a head-on collision?

(b) Find the time and place at which P_1 changes direction for the first time. Give exact answers (in terms of π, e, square roots, logs, etc.) and also give 2-place decimal approximations. Then do the same for P_2.

3 (X). In this problem you will consider two families of curves. **(a)** In this part, the first family consists of the curve $xy^2 = 1$, the curve $xy^2 = 2$, etc., that is, the curves $xy^2 = C$, one for each constant C. Pick a few values of C and sketch the curves. The second family consists of the curves $2x^2 - y^2 = C$, again one for each constant C. Sketch a few of these too, in the same window.

(b) Continuing with Part (a), show that any curve in the first family and any curve in the second family are perpendicular wherever they happen to meet. For this reason the two families are said to be "orthogonal". Orthogonal families play a very important role in the study of electromagnetism and heat flow. Besides, they make elegant pictures.

(c) Now consider a different family of curves, the one defined by the equations $xy^3 = C$. Can you find a family orthogonal to it? (*Hint:* if you choose the right constant A, the family $Ax^2 - y^2 = C$ will work.) Check your answer with a few graphs.

4 (X X). Let a be a positive constant and consider the functions

$$f(x) = \arcsin\left(\frac{x}{a}\right) \text{ and } g(x) = a \arctan\left(\frac{x}{a}\right).$$

Find the derivatives of f and g and express them in as simple a form as possible. There is a certain value of a for which the lines tangent to the graphs of these two functions at $x = 1$ are parallel lines. Find that value of a to 3-place accuracy. (Find an exact equation satisfied by a, and then get an accurate enough solution from your calculator.)