1. (2 pts) Convince me, using an epsilon-delta argument, that \( \lim_{x \to 3} (2x + 1) \neq 6. \)

\[
|2x + 1 - 6| < \varepsilon
\]

\[-\varepsilon < 2x - 5 < \varepsilon\]

\[
\frac{5 - \varepsilon}{2} < x < \frac{5 + \varepsilon}{2}
\]

For \( \varepsilon \) really small, there are numbers centered around \( \frac{5}{2} = 2.5 \), say \((2.4, 2.6)\), which cannot happen if we demand that \( 0 < |x - 3| < 8 \).

For this would be \( x \in (3 - 8, 3 + 8) \). In particular, \( x \) larger than \( 3 \) should also satisfy \( f(x) \) is \( \varepsilon \)-close to 6. Impossible.

2. (4 pts) If \( \lim_{x \to 1} f(x) = 3 \) and \( \lim_{x \to 3} g(x) = 1 \), then compute \( \lim_{x \to 1} \left( 10 + g(f(x)) - f(x) \right)^{1/3} \).

(Assume \( f \) is continuous at \( x = 1 \) and \( g \) is continuous at \( x = 3 \).)

\[
\lim_{x \to 1} \left( 10 + g(f(x)) - f(x) \right)^{1/3} = \left( \lim_{x \to 1} 10 + \lim_{x \to 3} g(f(x)) - \lim_{x \to 1} f(x) \right)^{1/3}
\]

\[
= \left( 10 + g \left( \lim_{x \to 1} f(x) \right) - 3 \right)^{1/3}
\]

\[
= \left( 10 + g(3) - 3 \right)^{1/3} = \left( 10 + 1 - 3 \right)^{1/3}
\]

\[
= 8^{1/3} = 2
\]
3. (4 pts) Find values of $a$ and $b$ so that the function given by

$$f(x) = \begin{cases} \frac{ax^2}{x} & x \leq -2 \\ bx - a & -2 < x \leq 0 \\ \frac{\tan x}{x} & 0 < x < \pi/2 \end{cases}$$

is continuous at every point in its domain. (Use theorems and observations from class to evaluate any limits involved, not epsilon-delta arguments.)

Need:

1. $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x) = f(-2)$
2. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

1. $a(-2)^2 = b(-2) - a$
2. $b(0) - a = \lim_{x \to 0^+} \tan x = \lim_{x \to 0^+} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = \left(\lim_{x \to 0^+} \sin x / x \right) \left(\lim_{x \to 0^+} \cos x / x \right) = 1 \cdot 1 = 1$

$\therefore \ a = -1$

$1 + 2 \Rightarrow -4 = -2b + 1 \Rightarrow b = -2.5 = -\frac{5}{2}$

For all other points within $(-\infty, \pi/2)$, $f(x)$ is continuous by theorems from class. (polys. + quotients of cont. funcs. are cont.)