Groupwork #1, Math 162 (Spring 2011)

Write up your solution to ONE of these problems (announced in class and on the course web page) and submit it on Thursday. Your solution should follow the Groupwork Write-up Guidelines & Hints (available on the course web page) as well as the ground rules laid out in the syllabus.

1 (Warm-up). Suppose that \( f \) is a continuous function (defined on \( \mathbb{R} \)) and that it is known that
\[
\int_{0}^{1} f(x) \, dx = 5, \quad \int_{-1}^{0} f(x) \, dx = 3, \quad \int_{0}^{2} f(x) \, dx = 8 \quad \text{and} \quad \int_{0}^{4} f(x) \, dx = 11.
\]
Evaluate the integrals:
\[
\begin{align*}
(a) & \int_{0}^{2} f(2x) \, dx & (b) & \int_{0}^{\pi} \sin x f(\cos x) \, dx & (c) & \int_{2}^{3} xf(8 - x^2) \, dx
\end{align*}
\]

2 (Sketch this Lozenge). Let \( R \) be the parabolic region in the \( x-y \) plane bounded below by the curve \( y = x^2 \) and above by the line \( y = 1 \).
(a) Sketch \( R \). Set up and evaluate an integral that gives the area of \( R \).
(b) Suppose a solid has base \( R \) and the cross-sections of the solid perpendicular to the \( y \)-axis are squares. Sketch the solid and find its volume.
(c) Suppose a solid has base \( R \) and the cross-sections of the solid perpendicular to the \( y \)-axis are equilateral triangles. Sketch the solid and find its volume.

3 (Sketch this doughnut). Start with the region \( A \) in the first quadrant enclosed by the \( x \)-axis and the parabola \( y = 2x(2-x) \). Then obtain solids of revolution \( S_1, S_2, \) and \( S_3 \) by revolving \( A \) about the lines \( y = 4, y = -2, \) and \( x = 4 \), respectively. All three solids are (unusual) “doughnuts” which are 8 units across, whose hole is 4 units across, and whose height is 2 units. Sketch them.
(a) Which do you expect to have larger volume, \( S_1 \) or \( S_2 \)? Compute their volumes exactly.
(b) Compute the volume of \( S_3 \). (It may be harder to guess how \( V_3 \) compares to \( V_2 \) and \( V_1 \).)

4 (How do you slice it?). Compute the shaded area using \textit{“fnInt(”} on your calculator:
\[
\frac{x^2}{3} + y^2 = 1 \quad \text{and} \quad x^2 + \frac{y^2}{3} = 1
\]

5 (Just over half a tank). An oil tank has the shape of a cylinder with diameter 4 ft. It is mounted so that the axis of the cylinder is horizontal (the circular cross-sections of the cylinder are vertical). If the depth of the water is 2.5 ft., what percentage of the total capacity of the tank is filled? Draw a picture, setting up this problem, then solve it two ways:
(a) Use elementary geometry (compare areas of circular sectors).
(b) Express the answer in terms of a definite integral, then obtain an approximate numerical value for the integral using \textit{“fnInt(”} on your calculator.