1. (5 pts) Which of these statements are true and which are false? 

**Briefly justify each answer.** (E.g., limit calculations on the little-ooh questions, or giving an $M$—and an $x$ after which the $M$ holds—for the big-ooh questions.)

(a) $\ln x = o(e^x)$  \((Maybe \ l’Hôpital?)\)

True. $\lim_{x \to \infty} \frac{\ln x}{e^x} = \lim_{x \to \infty} \frac{1}{xe^x} = 0$.

(b) $\ln(1 + x) = O(e^x)$ \((Recall \ that \ the \ linearizations \ of \ these \ functions \ are \ x \ and \ 1 + x, \ respectively. \ Then \ you’ll \ need \ a \ concavity \ argument.)\)

True, since little-ooh implies big-Oh. (Apply Part (a).) Next I try to find an $M$ and an $X$ so that for $x > X$, the ratio is smaller than $M$: $\frac{\ln(1+x)}{e^x} < \frac{x}{1+x}$ (for all $x \geq 0$), since logarithms are concave down and exponentials are concave up (and both are increasing). Now, this ratio is clearly bounded above by $M = 2$ for any $x \geq 0$, so we’re done.

(c) $e^x = O(\ln x)$

False. If it were true, then $e^x$ and $\ln x$ would have the same order (after Part (b)). Thus the limit of the ratio $\frac{e^x}{\ln x}$ should go to some constant $0 < L < \infty$, which is absurd.

(d) $e^x = o(e^{x^2})$

True. $\lim_{x \to \infty} \frac{e^x}{e^{x^2}} = \lim_{x \to \infty} \frac{1}{xe^{x^2}} = 0$.

(e) $e^{x^2} = O(e^x)$ \((Try \ using \ M = e^m.)\)

False. Putting $M = e^m$, we need to compare $\frac{e^{x^2}}{e^x} < e^m$, or $e^{x^2} < e^{x+m}$. No way, because $x^2 - x$ is certainly not bounded above by any number $m$. 

OVERLEAF
2. (5 pts) If \((\sec x) \frac{dy}{dx} = e^{y+\sin x}\) and \(y(0) = \ln 2\), find an equation that (implicitly or explicitly) defines \(y\) as a function of \(x\). Show your work.

This is a separable differential equation. So we have:

\[
\int e^{-y} \, dy = \int \cos x e^{\sin x} \, dx
\]

\[-e^{-y} = e^{\sin x} + C\]

\[-1/2 = 1 + C\]

\[e^{-y} = 3/2 - e^{\sin x}.\]