18.701 Practice Quiz 2

1. Let $V$ be the real vector space whose elements are the polynomials of degree $\leq 4$, and let $W = \mathbb{R}^2$. Let $T : V \to W$ be the linear transformation defined by $T(f) = (f(2), f'(2))^t$, where $f'$ denotes the derivative. Determine the dimension of the kernel (the nullspace) of $T$.

2. As usual, $\rho_\theta$ stands for the operator of rotation of the plane through the angle $\theta$ about the origin, and $r$ is reflection about the horizontal axis.
   (a) Determine the matrix of the composed linear operator $m = r \rho_\theta$.
   (b) Geometrically, $m$ is reflection about a line. Determine this line.
   (c) What are the eigenvalues of $m$?
   (d) Is $m$ a diagonalizable operator?

3. The rotation through the angle $\frac{\theta}{2}$ about the point $(1,2)$ can be written in the form $t_v \rho_\theta$, where $t_v$ is translation by the vector $v$. Determine $v$ and $\theta$.

4. The figure below depicts part of a pattern $F$ that covers the plane $\mathbb{R}^2$. Let $G$ be the group of symmetries of $F$.
   (a) Determine the point group of $G$.
   (b) Let $T_G = T \cap G$ be the subgroup of translations in $G$. Determine the index of $T_G$ in $G$.

5. Let $G$ be the group of symmetries of a regular tetrahedron $T$, including the orientation-reversing symmetries.
   (a) Decompose the set of faces of $T$ into orbits, and describe the stabilizer of a face.
   (b) Determine the order of $G$.

6. Let $G$ be a group of order 20 whose center is the trivial group $\{1\}$. Let $x$ be an element of $G$ of order 4. What can you say about the order of the conjugacy class of $x$?